- $\mathcal{L}(f)(s) = \int_{0^-}^{\infty} e^{-st} f(t)\,dt$

- $f(\cdot)$ is of exponential order

if $\exists\ M, T, \sigma$ real, $\forall t > T$

$$|f(t)| < M e^{\sigma t}$$

- The Abscissa of convergence of $f(\ )$

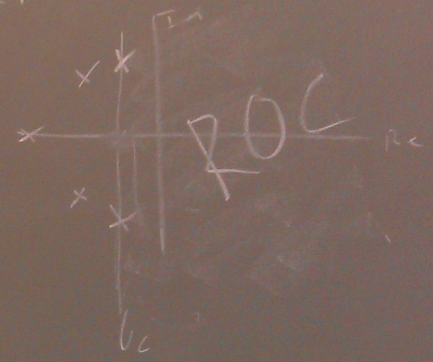
$$\sigma_c = \inf\{\sigma : \exists\ M, T, \forall t > T,\ |f(t)| < M e^{\sigma t}\}$$

Ex 7

If $f(t) = \dfrac{a(t)}{b(t)}$

poles of $f(t)$ are roots of

i.e. $t : b(t) = 0$



$r(t)dt$

cial order

l, $\forall t > T$

$< M e^{bt}$

ordergence of $f(\,)$

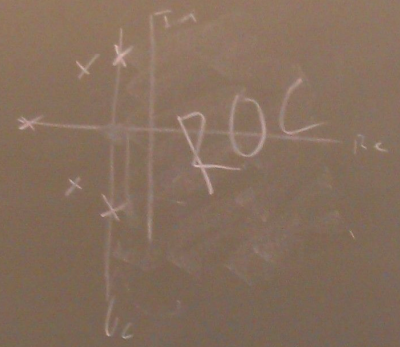
T, $\forall t > T$, $|f(t)| < M e^{bt}$

Ex 7

If $f(t) = \dfrac{a(t)}{b(t)}$

poles of $f(t)$ are roots of $b(t)$.

ie. t : $b(t) = 0$



Thm 3:

ROC is $Re(s) > b_c$

Proof:

$$|\hat{f}(s)| = \left| \int_{0^-}^{\infty} e^{-st} f(t)\,dt \right|$$

$$\leq \int_{0^-}^{\infty} |e^{-st}|\,|f(t)|\,dt$$

$$= \int e^{-bt} \underbrace{e^{-i\omega t}}_{1} |f(t)|\,dt$$

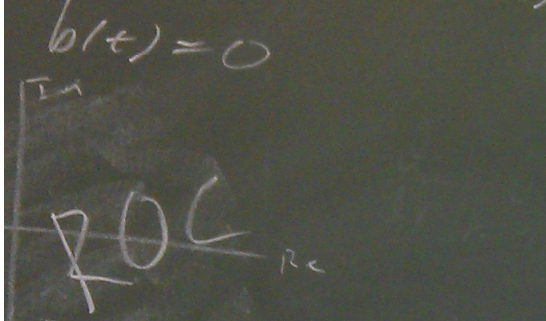
$$\leq M \int e^{-bt} e^{-bt}\,dt = M \int e^{-(b-s)t}\,dt$$

$b - s > 0$

$b(t)$

$f(t)$ are roots of $b(t)$.

$b(t) = 0$



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$b - b' > 0$

$$\frac{p(s)}{q(s)} = \frac{p_m s^m + \dots + p_1 s + p_0}{q_n s^n + \dots + q_0}$$

strictly proper: $m < n$.

If <u>not</u> strictly proper

$$f(s) = r(s) + \frac{v(s)}{q(s)}$$

$$= \sum_{i=1}^{K} \sum_{k=1}^{m_i} \frac{A_{ik}}{(s-s_i)^k}$$

$$K \leq n$$

# of distinct roots.

Ex 3

$$1 = (A_{11} + A_{21})s + (2A_{11} + A_{21})$$

Ex 4

$$(s - \zeta_i)(s - \bar{\zeta}_i) = s^2 + a_i s + b_i$$

$$\dots\dots\dots + \quad +$$

The Example of

$h(t)$ that is not of rational, $H(s)$.

$h(t) = \delta(t-\theta)$

pure delay of θ.

$$\boxed{\int_{-\infty}^{\infty} \delta(\tau)\phi(\tau)\,d\tau = \phi(0)}$$

$$H(s) = e^{-\theta s}$$

$$= \int_{0^-}^{\infty} \delta(t-\theta) e^{-s\tau}\,dt$$

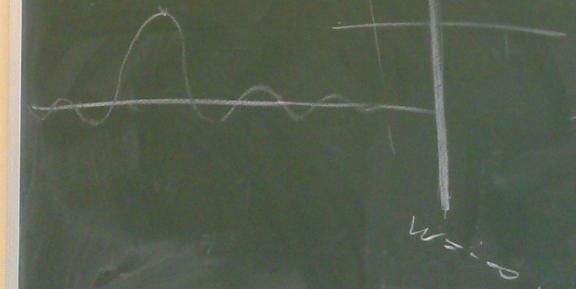
$$= \int_{-\theta^-}^{\infty} \delta(t) \boxed{e^{-s(t+\theta)}}\,dt = e^{-s\theta}$$

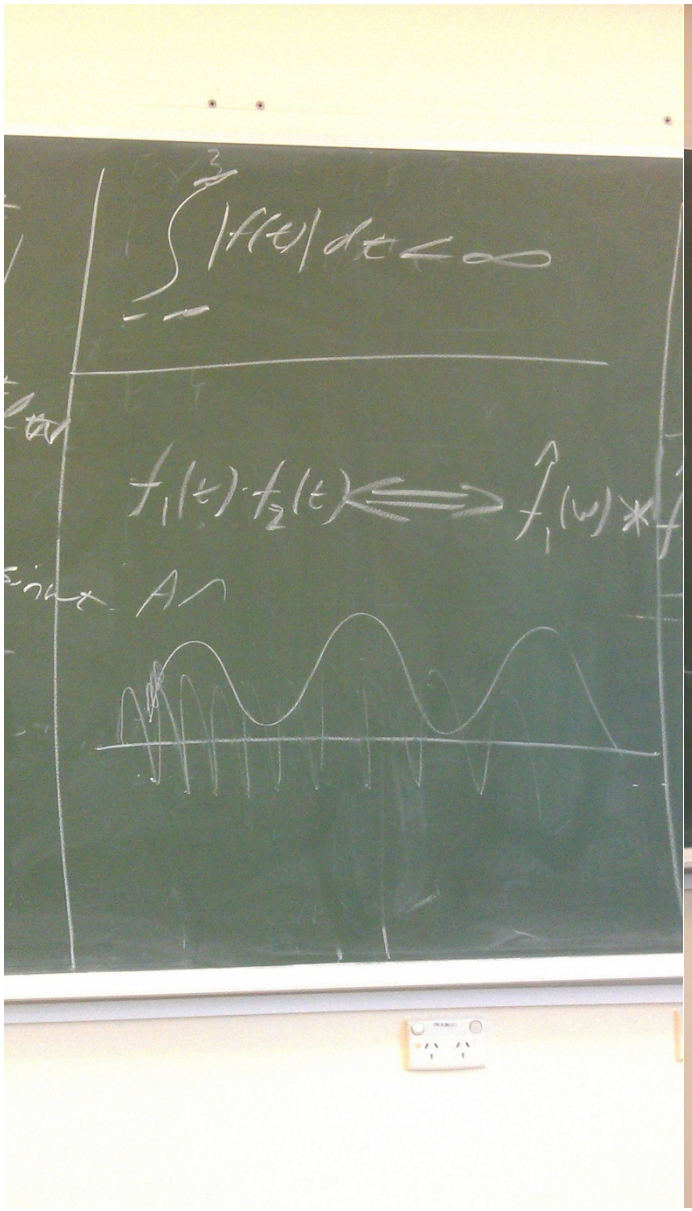
$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\,dt$$

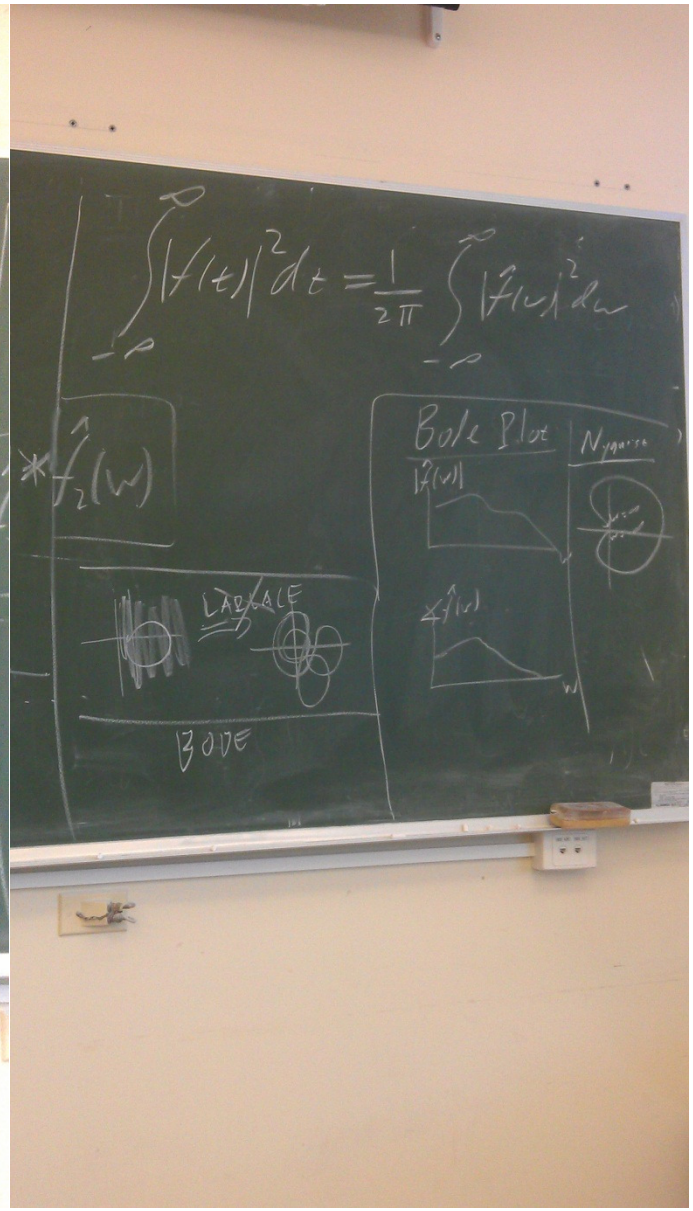
$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega t}\,d\omega$$

$$\hat{f}(\omega) = \cos\omega t + i\sin\omega t$$

Ex 16




$\int_{-\infty}^{\infty} |f(t)|\,dt < \infty$
$f_1(t) \cdot f_2(t) \Longleftrightarrow \hat{f}_1(\omega) * \hat{f}$


$\int_{-\infty}^{\infty} |f(t)|^2\,dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\hat{f}(\omega)|^2\,d\omega$
$* \hat{f}_2(\omega)$
LAPLACE
BODE
Bode Plot
Nyquist
$|\hat{f}(\omega)|$
$\angle \hat{f}(\omega)$

Ex 5:

$\Delta_T(t)$

T

$f_S(t) = f(t) \cdot \Delta_T(t)$

continuous signal

Fourier

$\hat{f}(\omega) * \hat{\Delta}_T(\omega)$

$|\hat{f}(\omega)|$

$\Delta_T(t)$

$f_{Filter}(\omega) =$

$$1(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \le t \end{cases}$$

$$\int_{-\infty}^{\infty} 1(t)\,\phi(t)\,dt = \int_{0}^{\infty} \phi(t)\,dt$$

$$\int_{-\infty}^{\infty} \dot{1}(t)\,\phi(t)\,dt = -\int_{-\infty}^{\infty} 1(t)\,\dot{\phi}(t)\,dt$$

Show: $\dot{1}(t) = \delta(t)$

$$\int_{-\infty}^{\infty} \dot{1}(t)\,\phi(t)\,dt = -\int_{-\infty}^{\infty} 1(t)\,\dot{\phi}(t)\,dt$$

$$= -\int_{0}^{\infty} \dot{\phi}(t)\,dt = -\left(\phi(\infty) - \phi(0)\right)$$

$$= \phi(0)$$

For inputs e^{st} the output is $H(s)e^{st}$

$\mathrm{Re}(s) = \sigma = 0$

often take $\cancel{|e^{st}| = 1}$

For inputs $e^{i\omega t}$

Output is

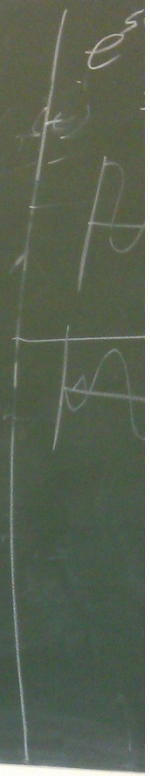
$$H(i\omega)e^{i\omega t}$$

$$y(t) = h(t) * u(t)$$

$$Y(s) = H(s)\,U(s)$$

$$\boxed{H(s) = \frac{Y(s)}{U(s)}}$$

e^{s}



$$e^{st} = e^{(\sigma + i\omega)t}$$
$$= e^{\sigma t} e^{i\omega t}$$

$$Y = UH$$
$$= EG_1H$$
$$= (R - Y_m)G_1H$$
$$= (R - Y G_2)G_1H$$

$$Y = R\,\boxed{\frac{G_1H}{1 + G_1G_2H}}$$