MATH4406 (Control Theory)
Unit 11: Closure

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With the exception of MPC, almost all of the formulations, methods and results that we studied predate 1970!!!

Yet it is only in the last couple of decades that Control Theory has become an active research field of its own, living in the intersection of analytic engineering and applied mathematics.

In that case, what else is out there in Control Theory? What methods have been developed since the 60's? What do current researchers do?

Subfields and Paradigms

- Non-linear (smooth) Control
- Non-linear (hybrid) Control
- Adaptive Control
- Robust Control
- Supervisory control
- Control of inherently stochastic systems
- Control of stochastic queueing networks (***)

Non-linear (smooth) Control

$$\begin{aligned} \dot{x}(t) &= f\big(x(t), u(t)\big) \\ y(t) &= g\big(x(t)\big) \end{aligned}$$

$f(\cdot)$ and/or $g(\cdot)$ are non-linear.

The concepts we have learned such as: Stability, Controllability, Observability, Feedback control, State Estimation and optimal control still have the same meaning. Yet results are more delicate and often depend on the structure of $f(\cdot)$ and $g(\cdot)$.

The mathematics involves Lie Algebras, manifolds and many tools from advanced analysis.

Non-linear (hybrid) Control

Hybrid dynamical systems have a continuous component $x(t)$ evolving in Euclidean space but also a discrete component $m(t)$ evolving on discrete set. Informally, for a given "mode" $m(t) = m$, $x(t)$ evolves according to the standard (say linear) dynamics that we know driven by A_m that depends on the mode:

$$\dot{x} = A_m x.$$

Then at the first time instance at which $x(t)$ reaches one of several sets, say $\mathcal{G}_{m'}$, the mode changes to m' and hence the dynamics change to,

$$\dot{x} = A_{m'} x.$$

E.g. a standard thermostat....

Here also, the same control questions exists (and have been partly answered): Stability, Controllability, Observability, Feedback control, State Estimation and optimal control.

Adaptive Control

Here the story (a very common one in practice) is the fact that the exact values of the plant parameters, say (A, B) are not known. Hence the parameters need to be estimated while the system is controlled (as opposed to off-line). In fact, some adaptive control techniques do **not** try to estimate the parameters, but simply try to control the system in an adaptive manner matching desired output to observed output and calibrating the control law on the go.

The theory is quite well developed, yet is advanced since typically **linear** plants controlled by adaptive controllers yield a **non-linear** systems.

Robust Control

Here the story is somewhat similar to adaptive control – there is plant uncertainty. Yet as opposed to developing a controller that tries to learn the plant, a controller is designed for the "worst case".

E.g. take an (A, B, C, D) system and assume that the actual A is $A + \delta G$ where G is some other matrix and δ is a scalar that is not too big.

A main theme is then to design a controller that ensures certain behavior (e.g. stability, optimality etc...) for a given range of δ.

Supervisory Control

This field uses a complete different set of tools: Computer science and discrete mathematics. The idea is to control discrete event systems with complicated (yet typically finite) state spaces. Think for example of a complicated photo-copier machine.

There are certain scientific questions dealing with state-reduction, computability and equivalent systems.

Control of inherently stochastic systems

The system

$$\dot{x} = Ax + \xi_x,$$

is inherently deterministic (e.g. A is modeled from Newton's laws) yet is subject to random disturbances.

Other systems arising in telecommunications, population models and logistics are well modeled as inherently stochastic systems (Markov Chains).

The field of Markov Decision Processes deals with finding optimal feedback laws for such systems – yet the problem is often with computation (curse of dimensionality).

Approximate dynamic programming for such systems is currently a hot research topic. Another related topic is stability analysis of such systems (e.g. Yoni's colloquium).

Control of stochastic queueing networks (***)