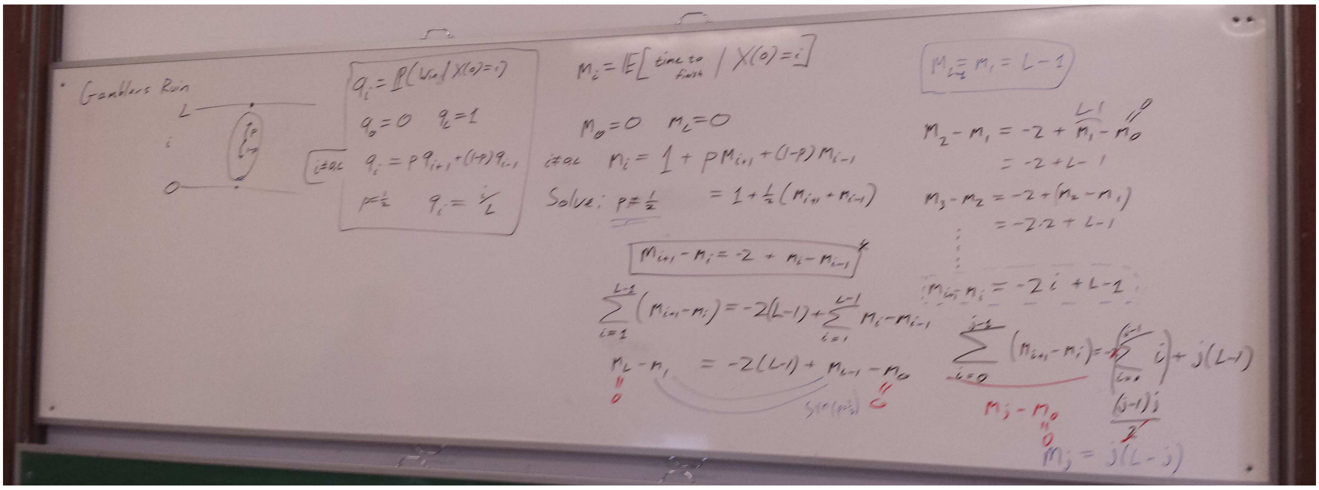
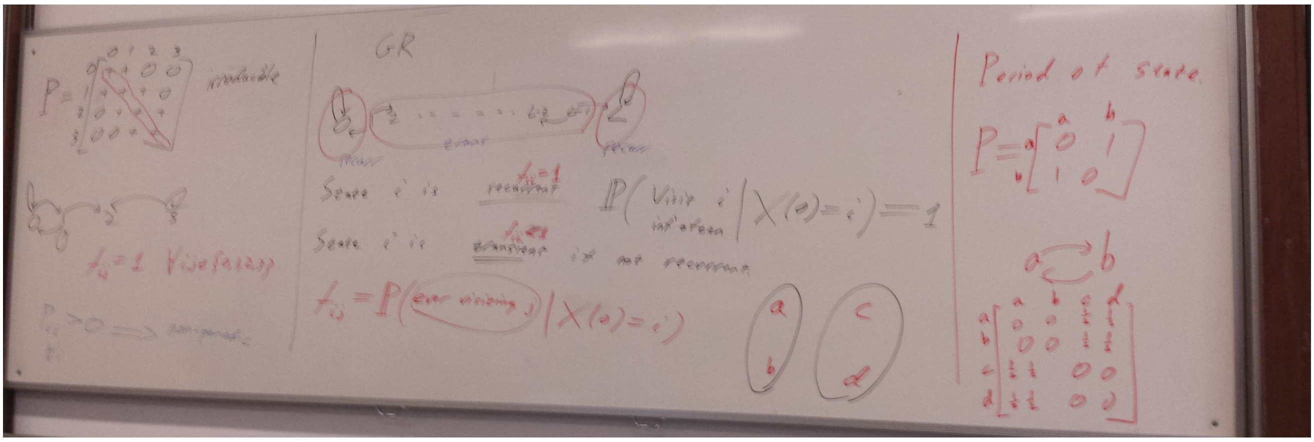
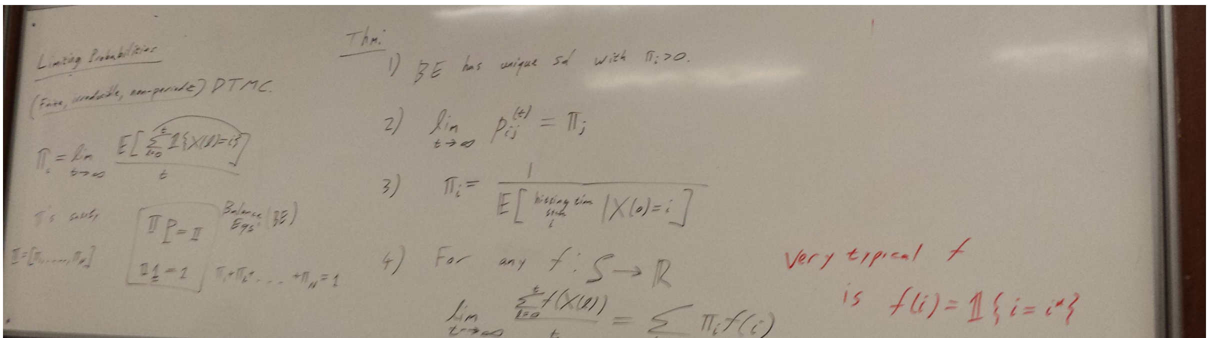

Gamblers Ruin
$q_i = \mathbb{P}(\text{win} \mid X(0)=i)$
$q_0 = 0 \quad q_L = 1$
$q_i = p\,q_{i+1} + (1-p)\,q_{i-1}$
$p = \frac{1}{2} \quad q_i = \frac{i}{L}$
$m_i = \mathbb{E}[\text{time to finish} \mid X(0)=i]$
$m_0 = 0 \quad m_L = 0$
$m_i = 1 + p\,m_{i+1} + (1-p)\,m_{i-1}$
Solve: $p = \frac{1}{2}$
$= 1 + \frac{1}{2}(m_{i+1} + m_{i-1})$
$m_{i+1} - m_i = -2 + m_i - m_{i-1}$
$m_{L-1} = m_1 = L-1$
$m_2 - m_1 = -2 + m_1 - m_0$
$= -2 + L - 1$
$m_{i+1} - m_i = -2i + L - 1$
$m_j = j(L-j)$

irreducible
GR
State i is recurrent
$\mathbb{P}(\text{Visit } i \text{ inf often} \mid X(0)=i) = 1$
State i is transient if not recurrent
$f_{ij} = \mathbb{P}(\text{ever visiting } j \mid X(0)=i)$
Period of state
a
b
c
d

Limiting Probabilities
(Finite, irreducible, non-periodic) DTMC.
Thm:
1) BE has unique sol with $\pi_i > 0$.
2) $\lim_{t\to\infty} p_{ij}^{(t)} = \pi_j$
3) $\pi_i = \frac{1}{\mathbb{E}[\text{hitting time} \mid X(0)=i]}$
4) For any $f: S \to \mathbb{R}$
$\lim_{t\to\infty} \frac{\sum_{t=0}^{t} f(X(t))}{t} = \sum_{i\in S} \pi_i f(i)$
Very typical f
is $f(i) = \mathbb{1}\{i = i^*\}$
$\pi P = \pi$
Balance Eqs (BE)
$\pi_1 + \pi_2 + \dots + \pi_N = 1$