Probability and Statistics
for Final Year Engineering Students

By Yoni Nazarathy, Last Updated: May 26, 2011.

Home Work Project #2

Due date: Monday June 13, 9:00 PM.

Please hand in in Yoni's mail box on the 7'th Floor of EN.

Hand in instructions: Please hand in a stapled hard-copy no later than the due date. Your hand in may be in a combination of handwriting and computer output. Make sure all your graphs are clear, well-scaled, and properly labeled. A portion of the grade will be based on the clarity of the presentation. Non-original work will not be graded. Hand in, in groups of one or two.

1) **Simple Stochastic Processes**

Let $X_1, X_2, \ldots$ be an infinite sequence of independent random variables, each having the same probability distribution with mean μ and variance σ^2. Let,

$$Y_n = \begin{cases} \sum_{i=1}^{n} X_i & n = 1,2,\ldots \\ 0 & n = 0 \end{cases}$$

A. What is the:
   i. Mean function, $m(n) = E[Y_n]$.
   ii. Variance function, $V(n) = Var(Y_n)$.
   iii. $Cov(Y_k, Y_l)$.

B. Let $X_i \sim Bin(1, p)$.
   i. Simulate a few illustrative trajectories of the process (for some arbitrary value of p that you select) and for n=0,…,100.
   ii. Simulate many trajectories to verify your result for Aiii above (by estimating the covariance).
   iii. Calculate: $P(X_{30} = 5, X_{40} = 7)$ for the arbitrary value of p that you select. Check that your result is correct by simulation.

C. Let $X_i \sim Uniform(0,2)$. Consider the stochastic process, $\tilde{Y}_n = \frac{Y_n - m(n)}{\sqrt{V(n)}}$.
   i. Simulate a trajectory of $\tilde{Y}_n$ for n=1,…,30.
   ii. Does $\tilde{Y}_n$ converge to a limiting distribution? What is this distribution? Use many simulations to show the distributions of $\tilde{Y}_n$ for n=1,2,3,4,5,10,20,30.

2) **The multi-dimensional normal distribution.**

A. Write the density function of the two dimensional normal distribution.

B. Show that the parameter ρ is indeed the correlation coefficient (calculate covariance, variance etc…).

C. Show that the two dimensional case is a special case of the n dimensional case (you will need to invert the 2x2 covariance matrix etc..).

D. One (popular) way to generate simultaneously a pair of independent Normal(0,1) random variables is to select a uniform angle in the **range** of $[0,2\pi]$ and to choose an independent **radius** according to the so-called Rayleigh distribution (look this distribution up on the web). This yields a polar representation of a coordinate, which when converted to Cartesian coordinates is distribution as two independent standard normal.
Generating a Rayleigh distributed random variable is easy by means of a transformation of a uniform (look it up). Use this method to generate a million random pairs and construct their 3 dimensional histogram, comparing the resulting shape to the two dimensional normal.

Good Luck.