Probability and Statistics
for Final Year Engineering Students

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Starred Exercises 3:
The Basics of Statistical Inference:
Point Estimation, Confidence Intervals and Hypothesis Testing

Starred Exercises:

1) Find an expression for the variance of the sample variance. I.e. find,
$$Var\left(\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}\right).$$
Use Monte-Carlo simulation to see if your result is correct (i.e. simulate sample variances and take the sample variance of the sample variance).
2) In deriving the confidence interval for the sample proportion, we went from:
$$P\left(\hat{p}-z_{1-\frac{\alpha}{2}}\frac{\sqrt{p(1-p)}}{\sqrt{n}} \le p \le \hat{p}+z_{1-\frac{\alpha}{2}}\frac{\sqrt{p(1-p)}}{\sqrt{n}}\right)=1-\alpha.$$
To
$$P\left(\hat{p}-z_{1-\frac{\alpha}{2}}\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} \le p \le \hat{p}+z_{1-\frac{\alpha}{2}}\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}\right)=1-\alpha.$$
Conduct numerical/simulation experiments to quantify the error in this approximation.

3) Power of a test: Consider a two sided hypothesis test for the population proportion. In this case the power of the test ($1-\beta$) is a function of the actual value of p (in H_1). Curves that show the power as a function of the unknown parameter (p in this case) are called **power-curves**. There is a different curve for each level of α and the sample size n. Plot a few of these curves on the same graph. Explain your results.