Statistics for Final Year Engineering Students
Intermediate Class Test, Monday May 9, 2011.
Lecturer: Dr. Yoni Nazarathy.

Duration: 100 Minutes.

Allowed: Non-Communicating calculators, double sided A4 reference sheet.

Not Allowed: Any other material, talking of any sort, looking side to side, passing items.
Students who violate this will be asked to leave immediately.

Write your answers CLEARLY in the answer boxes only. Show working only when required. You may use spare paper supplied during the test.

A normal distribution table is supplied.

The test is composed of 8 questions. 13 points per question (Maximal grade: 104).

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1) A school bus has capacity for 50 students. The mean weight of a student is 55 Kg and the standard deviation is 10 Kg.

    a. What is the probability that the total weight of students on a full school bus exceeds 2800 Kg?

Result:

    b. State the statistical assumptions required for your answer in (a) to hold.

Assumptions:

2) You use a computer to generate 10 independent uniform random variables in the range [0,1]. You then throw away any random variable that is greater than 4/5, keeping only the random variables that are less than or equal to 4/5. What is the probability that you are left with 8 or less numbers? Calculate a precise numeric answer.

Result:

3) A mini-school-bus contains 7 seats numbered 1 to 7. On the way to school it stops at 4 stations, picking up one student per station. Each student that enters the bus picks a random empty seat. What is the probability that upon arrival to the school there is exactly one empty odd numbered seat?

Result:

4) Let $c > 0$ and consider a random variable X that has the following density function:

$$f(x) = \begin{cases} c\,x & 0 \le x \le 2 \\ 0 & otherwise \end{cases}.$$

a) What is the value of c?

Result:

b) What is the expected value of X?

Result:

c) Write the CDF (Cumulative Distribution Function) of X (it should be defined for all real x).

Result:

5) Let $X_1, X_2, \ldots$ be a sequence of numbers. As you know, this is the formula for the sample variance:

$$S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2.$$

Write a formula representing S_{n+1}^2 as a function of $\sum_{i=1}^{n} X_i$, $\sum_{i=1}^{n} X_i^2$, X_{n+1} and n.
That is, find and expression for f() where,

$$S_{n+1}^2 = f(\quad \sum_{i=1}^{n} X_i\,,\ \sum_{i=1}^{n} X_i^2\,, X_{n+1},\ \ n\ \ ).$$

Result:

6) A random sample of 1000 red apples in a super market chain containes 12 rotten apples. An apprentice statistician at the super market chain uses the following formula for confidence intervals:

$$\hat{p} \pm z_{1-\frac{\alpha}{2}}\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}},$$

with $\alpha = 0.05$. Note that $z_{.975} = 1.96$.

The statistician reports to management that there is a 95% chance that the proportion of rotten apples in the whole super market chain is in the range [-0.008, 0.032].

Which of the following statements is correct?
(circle the correct answers – there may be more than one).

a) The statistician has made a calculation mistake in the above formula.

b) The confidence interval contains negative values because it uses the normal approximation to the binomial distribution and $\hat{p}$ is small while n is not "big enough".

c) Increasing the confidence level to a high enough level will ensure that the confidence interval is strictly positive.

d) Decreasing the confidence level to a low enough level will ensure that the confidence interval is strictly positive.

e) The confidence interval contains negative values because $\hat{p}$ is a biased estimator.

7) Weights of items shipped by a mail-order company are normally distributed with a mean of 20Kg and a variance of 4 Kg. Items weighing more than 22 Kilograms incur an additional cost to the company of 15$ per item. The company is shipping 100 items. What is the mean value of additional costs?

Result: Brief Explanation:

8) Let X_1 and X_2 be independent random variables each taking values in the sets $\{1, \dots, n\}$ with equal probability (e.g. If n=6 these are the results of independent die throws).
Find a simple formula in terms of n for the mean of $Y = X_1 + X_2$. Show your working.

Final Result: Working:

(Reminder: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$).

Good Luck.