Stochastic Modeling for Engineers
HW Project Number 2: Joint Distributions

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1. Do problem 5.129. Note that the joint CDF of X and Y (in case you did not study it), is defined as,
$$F_{X,Y}(x,y) = P\big(X \le x,\, Y \le y\big).$$

2. Write down the PDF of a bivariate (two-dimensional) normal distribution in terms of the parameters $\mu_i, \sigma_i,\ i = 1,2$ and ρ. Consider now the formula for a general multi-variate normal distribution in terms of the mean vector and covariance matrix. Show that it agrees with the bivariate formula. (This requires writing the covariance matrix in terms of $\sigma_i,\ i = 1,2$ and ρ and then manipulating the expressions).

3. Do problem 5.132.

4. A useful way of randomly generating normal random variables is by generating a pair using polar coordinates. For this choose a random angle $\theta \sim$ uniform $(0, 2\pi)$ and a radius according to the Rayleigh distribution – look it up in Wikipedia or elsewhere and see how to generate random samples. The resulting (angle,radius) pair is then converted from polar to cartesian (x,y) coordinates and is distributed like a bi-variate normal with mean $(0,0)$ and an identity covariance matrix. (If you don't understand look it up on the web or in the book).
   (a) Explain (understand) why the resulting method works.
   (b) For a single standard normal random variable, calculate (using a table or numeric integral), $P(1 \le Z \le 2)$.
   (c) Now estimate the above probability by generating $10,000$ normal random variables using the above method.
   (d) Assume you now want to generate bi-variate normal random variables with mean $(5, -2)$, variances 4 and 9 respectively, and correlation coefficient $= 1/2$. Find the transformation from the standard bi-variate normal that yields the desired distribution.
   (e) Use the above to estimate the probability of the random variable falling within a radius of 1 of its mean (using $10,000$ samples).
   (f) Compare the above to a numeric (2 dimensional) integral calculation.

Good Luck.