Stochastic Modeling for Engineers
HW Project Number 6: Optimum Linear Systems and the Kalman Filter

Yoni Nazarathy

November 3, 2011

1. Do problem 10.56.
2. Do problem 10.57.
3. Do problem 10.69.
4. Assume the monthly population of an animal species on an island, Z_n, follows the sequence

$$Z_n = a_{n-1} Z_{n-1} + W_{n-1}, \quad n = 1, 2, \ldots,$$

with $Z_0 = 100$, $a_n = \sqrt{n}$ and W_n a sequence of i.i.d. 0-mean Gaussian random variables with variance $\sigma_W^2 = 4$. Obviously Z_n is not an integer, yet use this as an approximation. Assume the population is measured every month, X_n, where,

$$X_n = Z_n + N_n,$$

and N_n is a sequence of i.i.d. 0-mean Gaussian random variables with variance $\sigma_N^2 = 9$.

- Generate and plot 3 random trajectories for $n = 0, 1, \ldots, 20$ of both X_n and Z_n.
- Write out equations (10.107) in the book for $n = 0, 1, \ldots, 3$ and solve them from $h_j^{(n)}$.
- Use the result above to estimate Z_n from X_n for the first observations (for the the random trajectories above).
- Implement the Kalman filter algorithm (page 621) for estimating Z_n based from X_n (for the random trajectories). Verify that your estimates agree with the results of equations (10.107) for the first observations.

Good Luck.