Class 4 : Programming in Mathematica Part #2

Note : Almost all of the examples below are taken from the Mathematica documentation center.

In this class we shall see how to handle, expressions, lists, functions, patterns (a bit). In the next class we shall see shall look at procedural programming in Mathematica.

Making Lists of Objects

```
{3, 5, 1}
```

```
{3, 5, 1}
```

```
{3, 5, 1}^2 + 1
```

```
{10, 26, 2}
```

```
{6, 7, 8} - {3.5, 4, 2.5}
```

```
{2.5, 3, 5.5}
```

```
Exp[%] // N
```

```
{12.1825, 20.0855, 244.692}
```

```
v = {2, 4, 3.1}
```

```
{2, 4, 3.1}
```

```
v / (v - 1)
```

```
     4
{2,  -, 1.47619}
     3
```

Getting Pieces of Lists

```
t = {a, b, c, d, e, f, g}
```

```
{a, b, c, d, e, f, g}
```

```
Last[t]
```

```
g
```

```
t[[3]]
```

```
c
```

```
t[[3 ;; 6]]
```

```
{c, d, e, f}
```

```
t[[{1, 4}]]

{a, d}

Take[t, 3]

{a, b, c}

Take[t, -3]

{e, f, g}

Take[t, {3, 7, 2}]

{c, e, g}

Rest[t]

{b, c, d, e, f, g}

Drop[t, 3]

{d, e, f, g}
```

Defining Functions

```
f[x_] := x^2

f[a + 1]

(1 + a)^2

f[4]

16

? f

Global`f

f[x_] := x^2

hump[x_, xmax_] := (x - xmax)^2 / xmax

2 + hump[x, 3.5]

2 + 0.285714 (-3.5 + x)^2

hump[x_, xmax_] := (x - xmax)^4

? hump

Global`hump

hump[x_, xmax_] := (x - xmax)^4

Clear[hump]
```

Applying Functions to Parts of Expressions

```
    Map[f, {a, b, c}]

    {f[a], f[b], f[c]}

    take2[list_] := Take[list, 2]

    Map[take2, {{1, 3, 4}, {5, 6, 7}, {2, 1, 6, 6}}]

    {{1, 3}, {5, 6}, {2, 1}}

    Map[f, a + b + c]

    f[a] + f[b] + f[c]

    Map[Sqrt, g[x^2, x^3]]
```

$$g\left[\sqrt{x^2}, \sqrt{x^3}\right]$$

```
    m = {{a, b}, {c, d}}

    {{a, b}, {c, d}}

    Map[f, m]

    {f[{a, b}], f[{c, d}]}

    Map[f, m, {2}]

    {{f[a], f[b]}, {f[c], f[d]}}

    Scan[Print, {a, b, c}]

a

b

c

    Scan[Print, 1 + x^2, Infinity]

1

x

2
```

x^2

Applying Functions to Lists and Other Expressions

```
    Apply[f, {a, b, c}]

    f[a, b, c]

    Apply[Times, {a, b, c}]

    a b c

    geom[list_] := Apply[Times, list]^(1 / Length[list])
```

```
Apply[List, a + b + c]

{a, b, c}

m = {{a, b, c}, {b, c, d}}

{{a, b, c}, {b, c, d}}

Apply[f, m]

f[{a, b, c}, {b, c, d}]

Apply[f, m, {1}]

{f[a, b, c], f[b, c, d]}

Apply[f, m, {0, 1}]

f[f[a, b, c], f[b, c, d]]
```

Adding, Removing and Modifying List Elements

```
Prepend[{a, b, c}, x]

{x, a, b, c}

Insert[{a, b, c}, x, 2]

{a, x, b, c}

Riffle[{a, b, c}, x]

{a, x, b, x, c}

?Riffle
```

Riffle[{e_1, e_2, …}, x] gives {e_1, x, e_2, x, …}.
Riffle[{e_1, e_2, …}, {x_1, x_2, …}] gives {e_1, x_1, e_2, x_2, …}.
Riffle[*list*, x, n] yields a list in which every n^{th} element is x.
Riffle[*list*, x, {i_{min}, i_{max}, n}] yields a list in which x appears if possible at positions i_{min}, $i_{min}+n$, $i_{min}+2n$, … , i_{max}. ≫

```
ReplacePart[{a, b, c, d}, 3 -> x]

{a, b, x, d}

ReplacePart[{{a, b}, {c, d}}, {1, 2} -> x]

{{a, x}, {c, d}}

v = {a, b, c, d}

{a, b, c, d}

v[[3]] = x

x

v

{a, b, x, d}

m = {{a, b}, {c, d}}

{{a, b}, {c, d}}
```

```
m[[All, 1]] = {x, y}; m
```

```
{{x, b}, {y, d}}
```

```
m[[All, 1]] = 0; m
```

```
{{0, b}, {0, d}}
```

Manipulating Lists by Their Indices

```
{a, b, c, d}[[{1, 3}]]
```

```
{a, c}
```

```
(m = {{a, b, c}, {d, e, f}, {g, h, i}}) // TableForm
```

```
a b c
d e f
g h i
```

```
m[[{1, 3}, {1, 2}]] // TableForm
```

```
a b
g h
```

```
Extract[m, {{1, 3}, {1, 2}}]
```

```
{c, b}
```

```
m = {{a[1], a[2], b[1]}, {b[2], c[1]}, {{b[3]}}};
```

```
Position[m, b[_]]
```

```
{{1, 3}, {2, 1}, {3, 1, 1}}
```

```
Extract[m, %]
```

```
{b[1], b[2], b[3]}
```

```
Insert[{{a, b, c}, {d, e}}, x, {2, 1}]
```

```
{{a, b, c}, {x, d, e}}
```

```
Delete[%, {2, 1}]
```

```
{{a, b, c}, {d, e}}
```

```
IdentityMatrix[3]
```

```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

```
ReplacePart[%, {2, 2} -> x]
```

```
{{1, 0, 0}, {0, x, 0}, {0, 0, 1}}
```

Nested Lists

```
Table[x^i + j, {i, 2}, {j, 3}]
```

$$\left\{\{1+x, 2+x, 3+x\}, \left\{1+x^2, 2+x^2, 3+x^2\right\}\right\}$$

```
Array[x^#1 + #2 &, {2, 3}]
```

$\{\{1+x,\ 2+x,\ 3+x\},\ \{1+x^2,\ 2+x^2,\ 3+x^2\}\}$

```
Table[x^i + j, {i, 3}, {j, i}]
```

$\{\{1+x\},\ \{1+x^2,\ 2+x^2\},\ \{1+x^3,\ 2+x^3,\ 3+x^3\}\}$

```
Array[a, {2, 3}]

{{a[1, 1], a[1, 2], a[1, 3]}, {a[2, 1], a[2, 2], a[2, 3]}}

Flatten[%]

{a[1, 1], a[1, 2], a[1, 3], a[2, 1], a[2, 2], a[2, 3]}

ArrayFlatten[{{{{1}}, {{2, 3}}}, {{{4}, {7}}, {{5, 6}, {8, 9}}}}]

{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}

Array[a, {2, 2, 2}]

{{{a[1, 1, 1], a[1, 1, 2]}, {a[1, 2, 1], a[1, 2, 2]}},
 {{a[2, 1, 1], a[2, 1, 2]}, {a[2, 2, 1], a[2, 2, 2]}}}

Transpose[%, {3, 1, 2}]

{{{a[1, 1, 1], a[2, 1, 1]}, {a[1, 1, 2], a[2, 1, 2]}},
 {{a[1, 2, 1], a[2, 2, 1]}, {a[1, 2, 2], a[2, 2, 2]}}}

m = {{{a, b}, {c, d}}, {{e, f}, {g, h}, {i}}};

Map[f, m, {2}]

{{f[{a, b}], f[{c, d}]}, {f[{e, f}], f[{g, h}], f[{i}]}}

Apply[f, m, {2}]

{{f[a, b], f[c, d]}, {f[e, f], f[g, h], f[i]}}
```

Pure Functions

```
h[x_] := f[x] + g[x]

Map[h, {a, b, c}]

{f[a] + g[a], f[b] + g[b], f[c] + g[c]}

Map[f[#] + g[#] &, {a, b, c}]

{f[a] + g[a], f[b] + g[b], f[c] + g[c]}

Function[x, x^2]
```

$\mathrm{Function}[x,\ x^2]$

```
%[n]
```

n^2

```
Map[Function[x, x^2], a + b + c]
```

$a^2 + b^2 + c^2$

```
Nest[Function[q, 1 / (1 + q)], x, 3]
```

$$\frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}$$

```
Function[{x, y}, x^2 + y^3][a, b]
```

$a^2 + b^3$

```
Map[#^2 &, a + b + c]
```

$a^2 + b^2 + c^2$

```
f[##, ##] &[x, y]
```

```
f[x, y, x, y]
```

```
Apply[f[##2, #1] &, {{a, b, c}, {ap, bp}}, {1}]
```

```
{f[b, c, a], f[bp, ap]}
```

```
fromdigits[digits_] := Fold[(10 #1 + #2) &, 0, digits]
```

```
fromdigits[{1, 5, 2, 3, 4}]
```

```
15 234
```

Applying Functions Repeatedly

```
? Fold
```

Fold[*f*, *x*, *list*] gives the last element of FoldList[*f*, *x*, *list*]. ≫

```
? FoldList
```

FoldList[*f*, *x*, {*a*, *b*, …}] gives {*x*, *f*[*x*, *a*], *f*[*f*[*x*, *a*], *b*], …}. ≫

```
? Nest
```

Nest[*f*, *expr*, *n*] gives an expression with *f* applied *n* times to *expr*. ≫

```
? NestList
```

NestList[*f*, *expr*, *n*] gives a list of the results of applying *f* to *expr* 0 through *n* times. ≫

```
Nest[f, x, 4]
```

```
f[f[f[f[x]]]]
```

```
NestList[f, x, 4]
```

```
{x, f[x], f[f[x]], f[f[f[x]]], f[f[f[f[x]]]]}
```

```
recip[x_] := 1 / (1 + x)
```

```
Nest[recip, x, 3]
```

$$\frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}$$

```
newton3[x_] := N[1 / 2 (x + 3 / x)]

NestList[newton3, 1.0, 5]

{1., 2., 1.75, 1.73214, 1.73205, 1.73205}

FixedPoint[newton3, 1.0]

1.73205

FixedPointList[newton3, 1.0]

{1., 2., 1.75, 1.73214, 1.73205, 1.73205, 1.73205}

divide2[n_] := n / 2

NestWhileList[divide2, 123 456, EvenQ]

{123 456, 61 728, 30 864, 15 432, 7716, 3858, 1929}

NestWhileList[newton3, 1.0, Unequal, 2]

{1., 2., 1.75, 1.73214, 1.73205, 1.73205, 1.73205}

NestWhileList[Mod[5 #, 7] &, 1, Unequal, All]

{1, 5, 4, 6, 2, 3, 1}

FoldList[f, x, {a, b, c}]

{x, f[x, a], f[f[x, a], b], f[f[f[x, a], b], c]}

Fold[f, x, {a, b, c}]

f[f[f[x, a], b], c]

FoldList[Plus, 0, {a, b, c}]

{0, a, a + b, a + b + c}
```

Testing and Searching List Elements

```
Position[{a, b, c, a, b}, a]

{{1}, {4}}

Count[{a, b, c, a, b}, a]

2

MemberQ[{a, b, c}, a]

True

MemberQ[{a, b, c}, d]

False
```

```
m = IdentityMatrix[3]
```

```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

```
FreeQ[m, 0]
```

```
False
```

```
Position[m, 0]
```

```
{{1, 2}, {1, 3}, {2, 1}, {2, 3}, {3, 1}, {3, 2}}
```

Building Lists from Functions

```
Array[p, 5]
```

```
{p[1], p[2], p[3], p[4], p[5]}
```

```
Table[p[i], {i, 5}]
```

```
{p[1], p[2], p[3], p[4], p[5]}
```

```
Map[p, Range[5]]
```

```
{p[1], p[2], p[3], p[4], p[5]}
```

$p[x\_] := p^2$

```
Array[p, 5]
Table[p[i], {i, 5}]
Map[p, Range[5]]
```

$\{p^2, p^2, p^2, p^2, p^2\}$

$\{p^2, p^2, p^2, p^2, p^2\}$

$\{p^2, p^2, p^2, p^2, p^2\}$

```
Array[# + #^2 &, 5]
```

```
{2, 6, 12, 20, 30}
```

```
Array[m, {2, 3}]
```

```
{{m[1, 1], m[1, 2], m[1, 3]}, {m[2, 1], m[2, 2], m[2, 3]}}
```

```
Array[Plus[##]^2 &, {3, 3}]
```

```
{{4, 9, 16}, {9, 16, 25}, {16, 25, 36}}
```

```
NestList[D[#, x] &, x^n, 3]
```

$\{x^n, n\, x^{-1+n}, (-1+n)\, n\, x^{-2+n}, (-2+n)\, (-1+n)\, n\, x^{-3+n}\}$

Selecting Parts of Expressions with Functions

```
Select[{2, 15, 1, a, 16, 17}, # > 4 &]
```

```
{15, 16, 17}
```

```
t = Expand[(x + y + z)^2]
```

$x^2 + 2\,x\,y + y^2 + 2\,x\,z + 2\,y\,z + z^2$

```
Select[t, FreeQ[#, x] &]
```

$y^2 + 2\,y\,z + z^2$

```
Select[{-1, 3, 10, 12, 14}, # > 3 &, 1]
```

```
{10}
```

Everything Is an Expression

```
x + y + z
```

```
x + y + z
```

```
FullForm[%]
```

```
Plus[x, y, z]
```

```
1 + x^2 + (y + z)^2
```

$1 + x^2 + (y + z)^2$

```
FullForm[%]
```

```
Plus[1, Power[x, 2], Power[Plus[y, z], 2]]
```

```
Head[f[x, y]]
```

```
f
```

```
?Head
```

Head[*expr*] gives the head of *expr*. ≫

```
Head[a + b + c]
```

```
Plus
```

```
Head[{a, b, c}]
```

```
List
```

```
Head[23 432]
```

```
Integer
```

```
Head[345.6]
```

```
Real
```

```
Part[{a, b, c}, 0]
```

```
List
```

```
Part[{a, b, c}, 1]
```

```
a
```

```
Part[{a, b, c}, 3]
```

```
c
```

```
Part[{a, b, c}, 4]
```

Part::partw : Part 4 of {a, b, c} does not exist. ≫

```
{a, b, c}⟦4⟧
```

```
Part[23 242, 0]
```

```
Integer
```

```
Part[23 242, 1]
```

Part::partd : Part specification 23242⟦1⟧ is longer than depth of object. ≫

```
23 242⟦1⟧
```

Immediate and Delayed Definitions

```
ex[x_] := Expand[(1 + x) ^2]
```

```
? ex
```

```
Global`ex
```

$ex[x\_] := Expand\left[(1 + x)^2\right]$

```
iex[x_] = Expand[(1 + x) ^2]
```

$1 + 2 x + x^2$

```
? iex
```

```
Global`iex
```

$iex[x\_] = 1 + 2 x + x^2$

```
ex[y + 2]
```

$9 + 6 y + y^2$

```
iex[y + 2]
```

$1 + 2 (2 + y) + (2 + y)^2$

```
r1 = RandomReal[]
```

```
0.0560708
```

```
r2 := RandomReal[]
```

```
{r1, r2}
```

```
{0.0560708, 0.6303}
```

```
{r1, r2}
```

```
{0.0560708, 0.359894}
```

Applying Transformation Rules

```
x + y /. x -> 3
```

```
3 + y
```

```
x + y /. {x -> a, y -> b}
```

a + b

```
x + y /. {{x -> 1, y -> 2}, {x -> 4, y -> 2}}
```

{3, 6}

```
Solve[x^3 - 5 x^2 + 2 x + 8 == 0, x]
```

{{x → -1}, {x → 2}, {x → 4}}

```
x^2 + 6 /. %
```

{7, 10, 22}

```
{x^2, x^3, x^4} /. {x^3 -> u, x^n_ -> p[n]}
```

{p[2], u, p[4]}

```
h[x + h[y]] /. h[u_] -> u^2
```

$(x + h[y])^2$

```
{x^2, y^3} /. {x -> y, y -> x}
```

$\{y^2, x^3\}$

```
x^2 /. x -> (1 + y) /. y -> b
```

$(1 + b)^2$

```
x^2 + y^6 /. {x -> 2 + a, a -> 3}
```

$(2 + a)^2 + y^6$

```
x^2 + y^6 //. {x -> 2 + a, a -> 3}
```

$25 + y^6$

```
log[a b c d] /. log[x_ y_] -> log[x] + log[y]
```

log[a] + log[b c d]

```
log[a b c d] //. log[x_ y_] -> log[x] + log[y]
```

log[a] + log[b] + log[c] + log[d]

THIS WAS A LOT OF MATERIAL - YOU MUST NOW PRACTICE.