תרגילי חזרה - אינטגרלים

$$\int_0^{\frac{\pi}{2}} \sin x \, dx = \qquad (1)$$

$$\int_0^{\infty} x e^{-\frac{x^2}{2}} \, dx = \qquad (2)$$

$$\int_0^{\infty} 4 e^{-2x} x^2 \, dx = \qquad (3)$$

$$\int_3^{\infty} \frac{81}{x^4} \, dx = \qquad (4)$$

$$\int_0^{\infty} \frac{1}{2} e^{-\frac{1}{2}x} \, dx = \qquad (5)$$

$$\int_{-\infty}^{\infty} \frac{1}{2} e^{-|x|} \, dx = \qquad (6)$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(x-3)^2}{2}} \, dx = \qquad (7)$$

$$\int_{-\infty}^{\infty} \frac{1}{3\sqrt{2\pi}} e^{-\frac{(x-4)^2}{18}} \, dx = \qquad (8)$$

$$\int_{x=0}^{1} \int_{y=0}^{2} \left(x^2 + \frac{xy}{3} \right) dy\,dx = \qquad (9)$$

$$\int_{y=0}^{2} \int_{x=0}^{2-y} 3x \, dx\,dy = \qquad (10)$$

$$\int_{x=0}^{\infty} \int_{y=0}^{\infty} \frac{e^{-\frac{x}{y}} e^{-y}}{y} \, dy\,dx = \qquad (11)$$

$$\int_{x=0}^{1} \int_{y=x}^{2x} \frac{32}{3} xy \, dy\,dx = \qquad (12)$$

פתרונות תרגילי חזרה באינטגרלים

$$\int_0^{\frac{\pi}{2}} \sin x \, dx = -\cos x \Big|_0^{\frac{\pi}{2}} = -(0-1) = 1 \qquad ①$$

$$\int_0^{\infty} x e^{-\frac{x^2}{2}} dx \underset{\substack{t=\frac{x^2}{2} \\ dt = x dx}}{=} \int_0^{\infty} e^{-t} dt = -e^{-t} \Big|_0^{\infty} = 1 \qquad ②$$

$$\int_0^{\infty} 4e^{-2x} x^2 dx = \qquad ③$$

$$\begin{bmatrix} u = x^2 & u' = 2x \\ v' = 4e^{-2x} & v = \frac{4e^{-2x}}{-2} = -2e^{-2x} \end{bmatrix}$$

$$= \underbrace{-2x^2 e^{-2x}}_{=0} \Big|_0^{\infty} + \int_0^{\infty} 4x e^{-2x} dx =$$

$$\begin{bmatrix} u = x & u' = 1 \\ v' = 4e^{-2x} & v = -2e^{-2x} \end{bmatrix}$$

$$= \underbrace{-2xe^{-2x}}_{=0} \Big|_0^{\infty} + \int_0^{\infty} 2e^{-2x} dx = \frac{2e^{-2x}}{-2} \Big|_0^{\infty} = -1[0-1] = 1$$

$$\int_{3}^{\infty} \frac{81}{x^4}\,dx = \frac{81\cdot x^{-3}}{-3}\Bigg|_{3}^{\infty} = -27\left[0-\frac{1}{27}\right] = 1 \qquad (4)$$

$$\int_{0}^{\infty} \frac{1}{2} e^{-\frac{1}{2}x}\,dx = \frac{1}{2}\cdot\frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}}\Bigg|_{0}^{\infty} = -\left[0-1\right] = 1 \qquad (5)$$

$$\int_{-\infty}^{\infty} \frac{1}{2}\cdot e^{-|x|}\,dx = \int_{0}^{\infty}\frac{1}{2}e^{-x}\,dx + \int_{-\infty}^{0}\frac{1}{2}e^{x}\,dx = \qquad (6)$$

$$= -\frac{1}{2}e^{-x}\Bigg|_{0}^{\infty} + \frac{1}{2}e^{x}\Bigg|_{-\infty}^{0} = -\frac{1}{2}(0-1)+\frac{1}{2}(1-0) = 1$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{(x-3)^2}{2}}\,dx \underset{\substack{y=x-3\\ dy=dx}}{=} \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{y^2}{2}}\,dy = \qquad (7)$$

$$= \underset{\text{הראינו בכתה.}}{\frac{1}{\sqrt{2\pi}}\cdot\sqrt{2\pi}} = 1$$

$$\int_{-\infty}^{\infty} \frac{1}{3\sqrt{2\pi}}\cdot e^{-\frac{(x-4)^2}{18}}\,dx = \int_{-\infty}^{\infty}\frac{1}{3\sqrt{2\pi}}\cdot e^{-\frac{\left(\frac{x-4}{3}\right)^2}{2}}\,dx = \qquad (8)$$

$$y = \frac{x-4}{3}$$

$$dy = \frac{1}{3}dx$$

$$= \int_{y=-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \underset{\text{הוכחנו בכיתה}}{=} \frac{1}{\sqrt{2\pi}} \cdot \sqrt{2\pi} = 1$$

(9)

$$\int_{x=0}^{1} \int_{y=0}^{2} \left(x^2 + \frac{xy}{3}\right) dy\, dx = \int_{x=0}^{1} x^2 y + \frac{xy^2}{6} \Big|_{y=0}^{2} dx =$$

$$= \int_{x=0}^{1} \left(2x^2 + \frac{2}{3}x\right) dx = \left(\frac{2x^3}{3} + \frac{\cancel{2}}{3} \cdot \frac{x^2}{\cancel{2}}\right)\Big|_{0}^{1} = \frac{2}{3} + \frac{1}{3} = 1$$

(10)

$$\int_{y=0}^{2} \int_{x=0}^{2-y} 3x\, dx\, dy = \int_{y=0}^{2} \frac{3x^2}{2} \Big|_{0}^{2-y} dy = \frac{3}{2} \int_{0}^{2} (2-y)^2\, dy =$$

$$= \frac{3}{2} \int_{0}^{2} (4 - 4y + y^2)\, dy = \frac{3}{2}\left[4y - \frac{4y^2}{2} + \frac{y^3}{3}\right|_{0}^{2} =$$

$$= \frac{3}{2}\left(8 - 8 + \frac{8}{3}\right) = 4$$

$$\int_{x=0}^{\infty}\int_{y=0}^{\infty} \frac{e^{-\frac{x}{y}} e^{-y}}{y}\, dy\, dx = \int_{y=0}^{\infty}\int_{x=0}^{\infty} \frac{e^{-y}}{y} \cdot e^{-\frac{x}{y}}\, dx\, dy = \qquad (11)$$

$$= \int_{y=0}^{\infty} \frac{e^{-y}}{y} \left.\frac{e^{-\frac{x}{y}}}{-\frac{1}{y}}\right|_{0}^{\infty} dy = \int_{y=0}^{\infty} -e^{-y}\left[0-1\right] dy =$$

$$= \int_{0}^{\infty} e^{-y}\, dy = \left.-e^{-y}\right|_{0}^{\infty} = -(0-1) = 1$$

$$\int_{x=0}^{1}\int_{y=x}^{2x} \frac{32}{3} xy\, dy\, dx = \int_{x=0}^{1} \left.\frac{32}{3} x \frac{y^2}{2}\right|_{x}^{2x} dx = \int_{0}^{1} \frac{16}{3} x \left(4x^2 - x^2\right) dx = \qquad (12)$$

$$= \int_{0}^{1} \frac{16}{3} \cdot 3x^3\, dx = \left.\frac{16x^4}{4}\right|_{0}^{1} = 4$$