$X \sim Bin(1000, \frac{1}{4})$ (א) (1)

$E(X) = 250$

$V(X) = 1000 \cdot \frac{1}{4} \cdot \frac{3}{4} = 187\frac{1}{2}$

נגדיר: $Y_i \sim Bernulli(\frac{1}{4})$ $i = 1, \dots, 1000$ בת"ל

$\Downarrow$

לכן: $X = \sum_{i=1}^{1000} Y_i$

$E(Y_i) = \frac{1}{4}$ $V(Y_i) = \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{16}$

לפי משפט הגבול המרכזי: $X \underset{\text{בקירוב}}{\sim} N(1000 \cdot \frac{1}{4}, 1000 \cdot \frac{1}{4} \cdot \frac{3}{4})$

$X \sim N(250, 187\frac{1}{2})$

$P(X < 100) \cong P\left(Z < \frac{100 - 250}{\sqrt{187\frac{1}{2}}}\right) = P(Z < -10.954) = 0$

(ב)

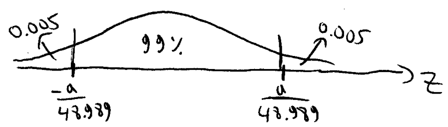
X - סכום הספרות

$Y_i = \begin{cases} 1 & 0.6 \\ 0 & 0.4 \end{cases}$ - ברנולי בת"ל $i = 1, \dots, 10{,}000$

$X = \sum_{i=1}^{10{,}000} Y_i \Rightarrow X \underset{\text{בקירוב}}{\sim} N(10{,}000 \cdot 0.6, 10{,}000 \cdot 0.6 \cdot 0.4) = N(6{,}000, 2400)$

$99\% = P(6000 - a < X < 6000 + a) = P\left(\frac{-a}{\sqrt{2400}} < Z < \frac{a}{\sqrt{2400}}\right) = P\left(\frac{-a}{48.989} < Z < \frac{a}{48.989}\right) =$

$= \Phi\left(\frac{a}{48.989}\right) - \Phi\left(\frac{-a}{48.989}\right)$



$\Phi\left(\frac{a}{48.989}\right) = 0.995 \Rightarrow \frac{a}{48.989} = 2.575$

$a = 126.146$

(2)

$X \sim Bin(n_1, p)$ X, y ב"ת

$y \sim Bin(n_2, p)$

$$M_{X+y}(t) = M_X(t) M_y(t) \underset{\text{ב"ת}}{=} (pe^t + q)^{n_1} (pe^t + q)^{n_2} = (pe^t + q)^{n_1 + n_2}$$

[illegible]

$$\Downarrow$$

$$X + y \sim Bin(n_1 + n_2)$$

(3)

ב"ת $X, y \sim G(p)$

$$M_{X+y}(t) = M_X(t) M_y(t) \underset{\text{ב"ת}}{=} \left(\frac{p \cdot e^t}{1 - qe^t}\right)\left(\frac{pe^t}{1 - qe^t}\right) = \left(\frac{p \cdot e^t}{1 - qe^t}\right)^2$$

[illegible]

$$\Downarrow$$

$$X + y \sim NB(2, p)$$

(4)

$y \sim N(20, 3)$ $X \sim N(10, 2)$

$cov(X, y) = 1$

ידוע שסכום של מ"מ נורמלים מתפלג נורמלית.

(א)

$$X + y \sim N(10 + 20,\ 2 + 3 + 2 \cdot 1) = N(30, 7)$$

$$p(X + y < 28) = p\left(Z < \frac{28 - 30}{\sqrt{7}}\right) = p(Z < -0.756) = 0.2266$$

(ב) שלילית

$$X \sim poiss(\lambda_1) \qquad Y \sim poiss(\lambda_2)$$ (5)

X ו Y ב"ת.

$$M_{X+Y}^{(t)} = M_X^{(t)} M_Y^{(t)}$$

$$M_X^{(t)} = E(e^{tX}) = \sum_{x=0}^{\infty} \frac{e^{tx} e^{-\lambda_1} (\lambda_1)^x}{x!} = e^{-\lambda_1} \sum_{x=0}^{\infty} \frac{(e^t \lambda_1)^x}{x!} =$$

$$= e^{-\lambda_1} e^{\lambda_1 e^t} = e^{-\lambda_1(1-e^t)}$$

$$M_Y^{(t)} = e^{-\lambda_2(1-e^t)}$$ מאותה סיבה.

$$M_{X+Y}^{(t)} = e^{-\lambda_1(1-e^t)} e^{-\lambda_2(1-e^t)} = e^{-(\lambda_1+\lambda_2)(1-e^t)}$$

$$\Downarrow$$

$$X+Y \sim poiss(\lambda_1+\lambda_2)$$

N_i - מס' הטלות שנפלו i $\qquad i=1,\dots,10$ (6)

$$p(i) = a^* \cdot i$$

$$p(N_1=n_1,\dots,N_{10}=n_{10}) = \frac{10!}{n_1!\cdots n_{10}!} \cdot (p_1)^{n_1} \cdots (p_{10})^{n_{10}}$$

א) $$p(N_1=1, N_2=1, \dots, N_{10}=1) = \frac{10!}{1!\,1!\,1!\cdots 1!} \cdot (a^* \cdot 1)^1 (a^* \cdot 2)^1 \cdots (a^* \cdot 10)^1 =$$

$$= (10!)^2 \cdot (a^*)^{10}$$

ב) $$p(N_5=8, N_{10}=2) = \frac{10!}{2!\,8!} (a^* \cdot 5)^8 (a^* \cdot 10)^2$$

(7)

$p_1 = 7\%$

$p_2 = 2\%$

$p_3 = 1\%$

$p_4 = 0.9$

$$p(N_1=10, N_2=5, N_3=5) = \frac{1000!}{10!\,5!\,5!\,980!} \cdot (0.07)^{10}(0.02)^{5}(0.01)^{5}(0.9)^{980}$$

(8) בינומי = מולטינומי עם 2 קבוצות.

(9)

N_1 - נק' 1 $\qquad p_1 = \frac{1}{2}$

N_2 - נק' 2 $\qquad p_2 = \frac{1}{4}$

N_3 - נק' 4 $\qquad p_3 = \frac{1}{8}$

N_4 - נק' 8 $\qquad p_4 = \frac{1}{16}$

N_5 - נק' 0 $\qquad p_5 = \frac{1}{16}$

א) $p(\text{0 נקודות}) = p(N_5=4) = \frac{4!}{4!\,0!\,0!\,0!\,0!}\left(\frac{1}{2}\right)^0\left(\frac{1}{4}\right)^0\left(\frac{1}{8}\right)^0\left(\frac{1}{16}\right)^0\left(\frac{1}{16}\right)^4$

ב) $p(\text{5 נקודות}) = p(N_1=3, N_2=1, N_5=0) + p(N_1=1, N_2=2, N_5=0) +$

$+ p(N_1=1, N_3=1, N_5=0) =$

$$= \frac{4!}{3!\,1!\,0!}\left(\frac{1}{2}\right)^3\left(\frac{1}{4}\right)^1\left(\frac{1}{16}\right)^0 + \frac{4!}{1!\,2!\,0!}\cdot\left(\frac{1}{2}\right)^1\left(\frac{1}{4}\right)^2\left(\frac{1}{16}\right)^0 + \frac{4!}{1!\,1!\,0!}\left(\frac{1}{2}\right)^1\left(\frac{1}{8}\right)^1($$

(10)

$X_1, \ldots, X_n \sim exp(2)$ ב"ת.

סך הרווח $= Y = 20 \sum_{i=1}^{n} X_i$

נגדיר $Z = \sum_{i=1}^{n} X_i$

$$M_Z^{(t)} = M_{\sum X_i}^{(t)} = M_{X_1}^{(t)} M_{X_2}^{(t)} \cdots M_{X_n}^{(t)} = \left(\frac{2}{2-t}\right)\left(\frac{2}{2-t}\right)\cdots\left(\frac{2}{2-t}\right) = \left(\frac{2}{2-t}\right)^n$$

$$\Downarrow$$

$$Z \sim erlang(n, 2) \qquad \left(f_Z^{(z)} = \begin{cases} \frac{e^{-2z} \, 2^n \, z^{n-1}}{\Gamma(n)} & z > 0 \\ 0 & \text{אחרת} \end{cases} \right)$$

$Y = 20 Z$

$$F_Y^{(y)} = P(Y \le y) = P(20Z \le y) = P\left(Z \le \frac{y}{20}\right) = F_Z^{\left(\frac{y}{20}\right)}$$

$$f_Y^{(y)} = \frac{1}{20} \cdot f_Z^{\left(\frac{y}{20}\right)} = \begin{cases} \frac{\frac{1}{20} \cdot e^{-\frac{2y}{20}} \, 2^n \left(\frac{y}{20}\right)^{n-1}}{\Gamma(n)} & y > 0 \\ 0 & \text{אחרת} \end{cases}$$

(11) (א)

$X_i \sim exp\left(\frac{1}{60}\right) \quad i = 1, \ldots, 100$

$$\bar{X}_{100} \sim N\left(\underset{E(X_i)}{60}, \underset{\frac{V(X_i)}{n}}{\frac{60^2}{100}}\right)$$

$$P(\bar{X}_{100} < 58) = P\left(Z < \frac{58-60}{\sqrt{\frac{60^2}{100}}}\right) = P(Z < 0.3\bar{3}) \approx 0.6293$$

(ב)

נגדיר: $\sum_{i=1}^{5} X_i = Y$

$$M_Y^{(t)} = M_{\sum X_i}^{(t)} = M_{X_1}^{(t)} \cdots M_{X_5}^{(t)} = \left(\frac{\frac{1}{60}}{\frac{1}{60} - t}\right)^5 \Rightarrow Y \sim erlang\left(5, \frac{1}{60}\right)$$

$$P(Y > 300) = 1 - \int_{y=0}^{300} \frac{e^{-\frac{1}{60}y} \left(\frac{1}{60}\right)^5 y^{5-1}}{\Gamma(5)} \, dy$$

(ג)

$$P\left(\sum_{i=1}^{5} X_i > 300\right) = P\left(Z > \frac{300-5\cdot 60}{\sqrt{5\cdot 60^2}}\right) =$$

$$= P(Z>0) = 0.5$$

(12)

$X,Y \sim U(0,1)$ בת"ל

$Z = X+Y$

ב.ל.

$$f_Z(z) = \begin{cases} z & 0<z<1 \\ 2-z & 1<z<2 \\ 0 & \text{אחרת} \end{cases}$$

$$M_Z(t) = M_{X+Y}(t) \underset{\text{בת"ל}}{=} M_X(t)\cdot M_Y(t) = \left(\frac{e^t-1}{t}\right)^2$$

נבדוק את $M_Z(t)$ עבור $f_Z(z)$

$$M_Z(t) = E\left(e^{zt}\right) = \int_{z=0}^{1} e^{zt}\cdot z\, dz + \int_{z=1}^{2} e^{zt}(2-z)\, dz =$$

$$\boxed{\begin{array}{l} \int_a^b e^{tz} z = \frac{z\cdot e^{tz}}{t}\Big|_a^b - \int_a^b \frac{e^{tz}}{t}dz \\ u=z \quad e^{tz}=v' \\ u'=1 \quad \frac{e^{tz}}{t}=v \end{array}}$$

$$= \frac{ze^{tz}}{t}\Big|_0^1 - \int_0^1 \frac{e^{tz}}{t}dz + \int_1^2 2e^{tz}dz - \left[\frac{ze^{tz}}{t}\Big|_1^2 - \int_1^2 \frac{e^{tz}}{t}dz\right] =$$

$$= \left(\frac{e^t}{t}-0\right) - \frac{e^{tz}}{t^2}\Big|_0^1 + \frac{2e^{tz}}{t}\Big|_1^2 - \left[\frac{2e^{2t}}{t} - \frac{e^t}{t} - \left(\frac{e^{tz}}{t^2}\Big|_1^2\right)\right] =$$

$$= \frac{e^t}{t} - \left(\frac{e^t}{t^2}-\frac{1}{t^2}\right) + \left(\frac{2e^{2t}}{t}-\frac{2e^t}{t}\right) - \left[\frac{2e^{2t}}{t}-\frac{e^t}{t}-\left(\frac{e^{2t}}{t^2}-\frac{e^t}{t^2}\right)\right] =$$

$$= \cancel{\frac{e^t}{t}} - \frac{e^t}{t^2} + \frac{1}{t^2} + \cancel{\frac{2e^{2t}}{t}} - \cancel{\frac{2e^t}{t}} - \cancel{\frac{2e^{2t}}{t}} + \cancel{\frac{e^t}{t}} + \frac{e^{2t}}{t^2} - \frac{e^t}{t^2} =$$

$$= \frac{-e^t+1+e^{2t}-e^t}{t^2} = \frac{(e^t)^2 - 2e^t + 1}{t^2} = \frac{(e^t-1)^2}{t^2}$$

מ.ש.ל.

(13)

$X \sim U(12,13)$ אורך מוט

בת"ת

$Y \sim U(10,15)$ רוחב מוט

א1) $E(X\cdot Y) \underset{\text{בת"ת}}{=} E(X)\cdot E(Y) = \left(\frac{12+13}{2}\right)\left(\frac{10+15}{2}\right) = 156.25$

ב) מולטינומי

$$\frac{100!}{10!\,3!\,20!\,67!}\left(\frac{1}{12}\right)^{10}\left(\frac{1}{12}\right)^{3}\left(\frac{1}{12}\right)^{20}\left(\frac{9}{12}\right)^{67}$$

פונק' הסתברות

$X\cdot Y =$

$y\cdot x$	
$10\cdot 12 = 120$	$1/12$
$10\cdot 13 = 130$	$1/12$
$15\cdot 12 = 180$	$1/12$
$15\cdot 13 = 195$	$1/12$
$11\cdot 12 = 132$	$1/12$
$11\cdot 13 = 143$	$1/12$
$12\cdot 12 = 144$	$1/12$
$12\cdot 13 = 156$	$2/12$
$13\cdot 12 = 156$	
$13\cdot 13 = 169$	$1/12$
$14\cdot 12 = 168$	$1/12$
$14\cdot 13 = 182$	$1/12$