פתרון תרגיל 8

1) א) $P(Z<\frac{1}{2}) = 0.6915$

ב) $P(Z<1.5) = 0.9332$

ג) $P(-3<Z<3) = P(Z<3) - P(Z<-3) = 0.9987 - 0.0013 = 0.9974$

ד) $P(-1.6<Z<3) = P(Z<3) - P(Z<-1.6) = 0.9987 - 0.0548 = 0.9439$

2) א) X - גובה באוכלוסיה $\quad E(X)=174 \quad V(X)=10^2$

$$P(153\le X\le 195) = P(-21 \le X-174 \le 21) = P(|X-E(X)| \le 21) \ge$$

$$\underset{\text{אי שוויון צ'בישב}}{\ge} 1 - \frac{10^2}{21^2} = 0.773$$

ב) $X \sim N(174, 10^2)$

$$P(153\le X\le 195) = P(-2.1 \le Z \le 2.1) = 0.9821 - 0.0179 = 0.9642$$

3) נתון: $X \sim \exp(\lambda)$

יהי: $Y = [X]$

$Y=0 \qquad 0 \le X < 1$

$Y=1 \qquad 1 \le X < 2$

$\vdots \qquad \vdots$

$$p(y=0) = p(0 \le x < 1)$$

$$p(y=1) = p(1 \le x < 2)$$

$$\vdots$$

$$p(y=k) = p(k \le x < k+1) = \int_k^{k+1} \lambda e^{-\lambda x}\, dx =$$

$$= -e^{-\lambda x}\Big|_k^{k+1} = e^{-\lambda k} - e^{-\lambda(k+1)} = e^{-\lambda k}\left(1-e^{-\lambda}\right) =$$

$$= \left(e^{-\lambda}\right)^k \left(1-e^{-\lambda}\right) \qquad k=0,1,2,\dots$$

$$\Downarrow$$

$$\underset{\text{סופר כשלונות}}{y} \sim G\left(1-e^{-\lambda}\right)$$

(4)

$X \sim exp(1)$ -שטח במ"ר

$y = \sqrt{x}$ -אורך דופן

$$p(y \le y) = p(\sqrt{x} \le y) = p(x \le y^2) = F_x(y^2)$$

$$f_y(y) = 2y \cdot f_x(y^2) = \begin{cases} 2y \cdot 1 \cdot e^{-1 \cdot y^2} & y > 0 \\ 0 & \text{אחרת} \end{cases}$$

$$p(y < 0.8) = \int_0^{0.8} 2y e^{-y^2}\, dy \underset{\substack{z=y^2 \\ dz=2y\,dy}}{=} \int_0^{0.64} e^{-z}\, dz = -e^{-z}\Big|_0^{0.64} = 1 - e^{-0.64} =$$

$$= 0.473$$

$y = F_X(x)$, X מ"מ רצ' (5)

$$F_Y(y) = P(Y \le y) = P(F_X(x) \le y) = P(X \le F_X^{-1}(y)) = F_X\left[F_X^{-1}(y)\right] = y$$

$$0 \le y \le 1$$

$$\Downarrow$$

$$Y \sim U(0,1)$$

$y = \ln x$ $\quad$ $X \sim U(0,1)$ (6)

$$F_Y(y) = P(Y \le y) = P(\ln x \le y) = P(X \le e^y) = F_X(e^y)$$

$$f_Y(y) = e^y f_X(e^y) = e^y \cdot 1 = \begin{cases} e^y & y<0 \\ 0 & \text{אחר} \end{cases}$$

$$F_X(t) = \int_{x=-\infty}^{t} f_X(x)\,dx = \int_{x=-\infty}^{t} f_X(-x)\,dx \underset{\substack{y=-x \\ dy=-dx}}{=} -\int_{\infty}^{-t} f_X(y)\,dy =$$ (א (7)

$$= \int_{-t}^{\infty} f_X(y)\,dy = P(X > -t) = 1 - F_X(-t)$$

$Z \sim N(0,1)$ (ב

$$f_Z(z) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}} \\ -\infty < z < \infty \end{cases} \qquad f_Z(-z) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{z^2}{2}} \\ -\infty < z < \infty \end{cases} \Rightarrow f_Z(z) = f_Z(-z)$$

$$\Downarrow$$

$$\Phi(z) = 1 - \Phi(-z)$$

(8)

I.Q ← רמת - $X \sim N(115, 12^2)$

$$p(X \geq 95) = p\left(Z \geq \frac{95-115}{12}\right) = 1-\Phi(-1.\overline{6}) = 0.9515$$

$$0.9515 \cdot 600 \approx 571$$

כלומר נצפה שיידחו $600-571 = 29$ סטודנטים.

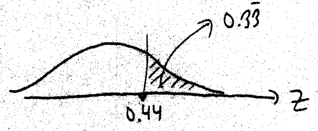
(9)

גובה הבנים - $X \sim N(175, 5^2)$

א) $p(X<170) = \Phi\left(\frac{170-175}{5}\right) = \Phi(-1) = 0.1587$

15.87%

ב)

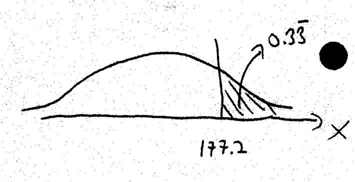


$$Z_{0.\overline{66}} = 0.44$$

$$0.\overline{66} = p(Z \leq 0.44) \Rightarrow 0.\overline{33} = p(Z > 0.44)$$

$$0.44 = \frac{X-175}{5}$$

$$X = 177.2$$



ג) $p\left(\text{בן כלשהו גבוה מ-185 ס"מ}\right) = p(X>185) = 1-\Phi\left(\frac{185-175}{5}\right) = 1-\Phi(2) =$

$$= 0.0228$$

$$y \sim Bin(20, 0.0228)$$

$$p(y \geq 2) = 1-p(y \leq 1) = 1-\left[\binom{20}{0} 0.0228^0\, 0.9772^{20} + \binom{20}{1} 0.0228^1\, 0.9772^{19}\right] =$$

$$= 0.0753$$

$$y = |x| \qquad X \sim N(\mu, \sigma^2) \qquad (10)$$

$$F_y(y) = P(Y \le y) = P(|X| \le y) = P(-y \le X \le y) = F_x(y) - F_x(-y)$$

$$f_y(y) = f_x(y) + f_x(-y) = \begin{cases} \dfrac{e^{-\frac{(y-\mu)^2}{2\sigma^2}} + e^{-\frac{(y+\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} & y > 0 \\ 0 & \text{אחרת} \end{cases}$$

$$X \sim N(1000, \sigma^2) \qquad (11$$

$$P(X \le 912) = 0.33$$

$$P\left(Z \le \frac{912-1000}{\sigma}\right) = 0.33 \qquad \frac{912-1000}{\sigma} = Z_{0.33} = -0.44$$

$$\Downarrow$$

$$\sigma = 200$$

$$P(850 \le X \le 950) = P(-0.75 \le Z \le -0.25) = \Phi(-0.25) - \Phi(-0.75) =$$

$$= 0.4013 - 0.2266 = 0.1747$$

$$y = x^2 \qquad X \sim N(\mu, \sigma^2) \qquad \text{נתון:} \qquad (12)$$

$$F_y(y) = P(X^2 \le y) = P(-\sqrt{y} \le X \le \sqrt{y}) = F_x(\sqrt{y}) - F_x(-\sqrt{y})$$

$$f_y(y) = \begin{cases} \frac{1}{2\sqrt{y}} \cdot f_x(\sqrt{y}) + \frac{1}{2\sqrt{y}} f_x(-\sqrt{y}) & y > 0 \\ 0 & \text{אחרת} \end{cases} =$$

$$= \begin{cases} \frac{1}{2\sqrt{y}} \left[\frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(\sqrt{y}-\mu)^2}{2\sigma^2}} + \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(-\sqrt{y}-\mu)^2}{2\sigma^2}} \right] & y > 0 \\ 0 & \text{אחרת} \end{cases} =$$

$$= \begin{cases} \frac{1}{2\sigma\sqrt{2\pi y}} \left(e^{-\frac{(\sqrt{y}-\mu)^2}{2\sigma^2}} + e^{-\frac{(\sqrt{y}+\mu)^2}{2\sigma^2}} \right) & y > 0 \\ 0 & \text{אחרת} \end{cases}$$