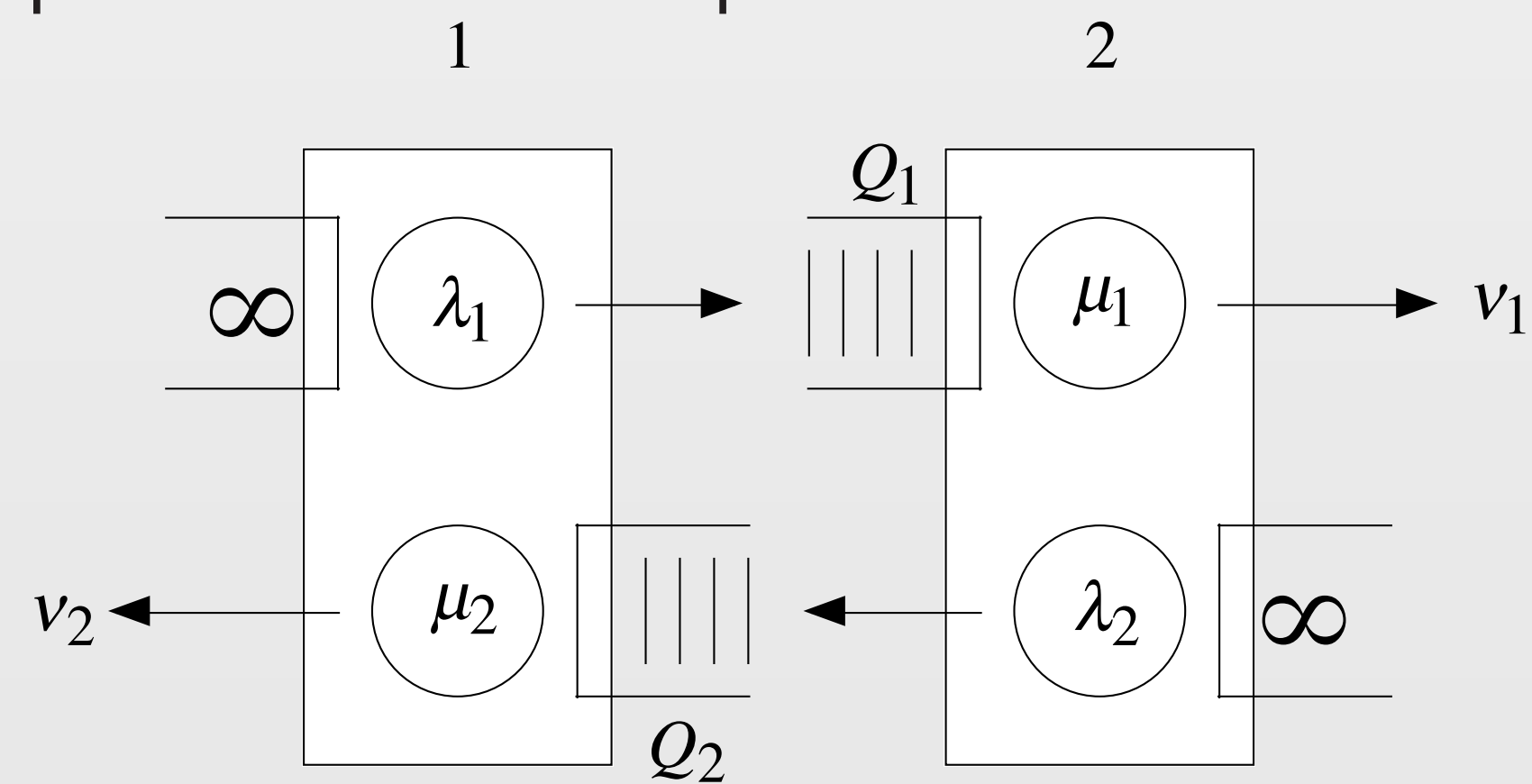


A Push Pull System: Infinite Supply of Work Allows Stable Full Utilization

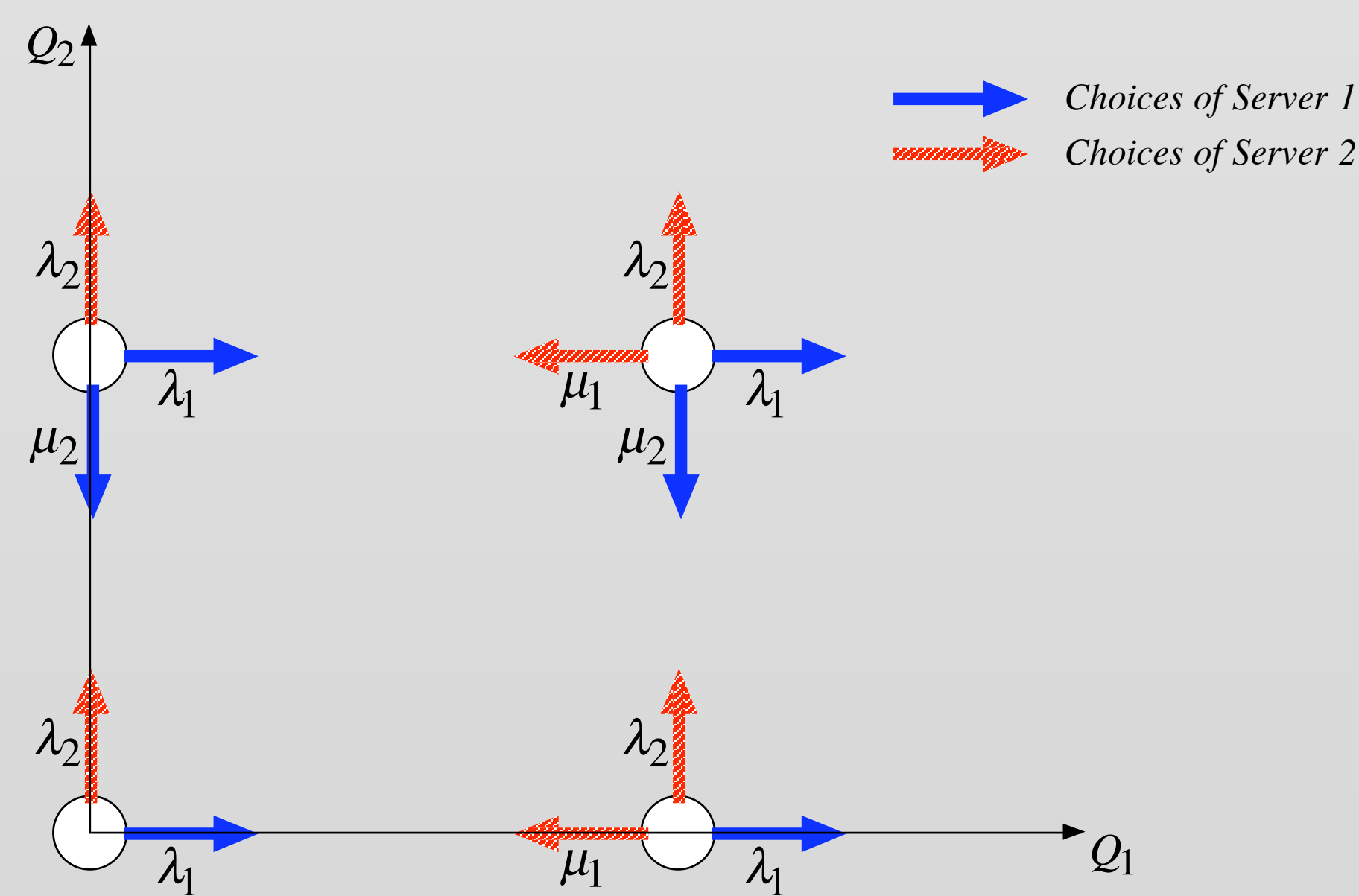
Anat Kopzon, Yoni Nazarathy, Gideon Weiss
Department of Statistics, The University of Haifa

Model

A queueing network with 2 servers, 2 types of jobs, 2 push activities with infinite supplies and 2 pull activities with queues.



- Rates for push activities: λ_i
- Rates for pull activities: μ_i
- $Q_i(t)$ = Queue sizes
- $D_i(t)$ = Cumulative departures
- Server may either push or pull (preemptive-resume)



- $\lambda_i < \mu_i$: Inherently stable case
- $\lambda_i > \mu_i$: Inherently unstable case

Goal: Full utilization and stability

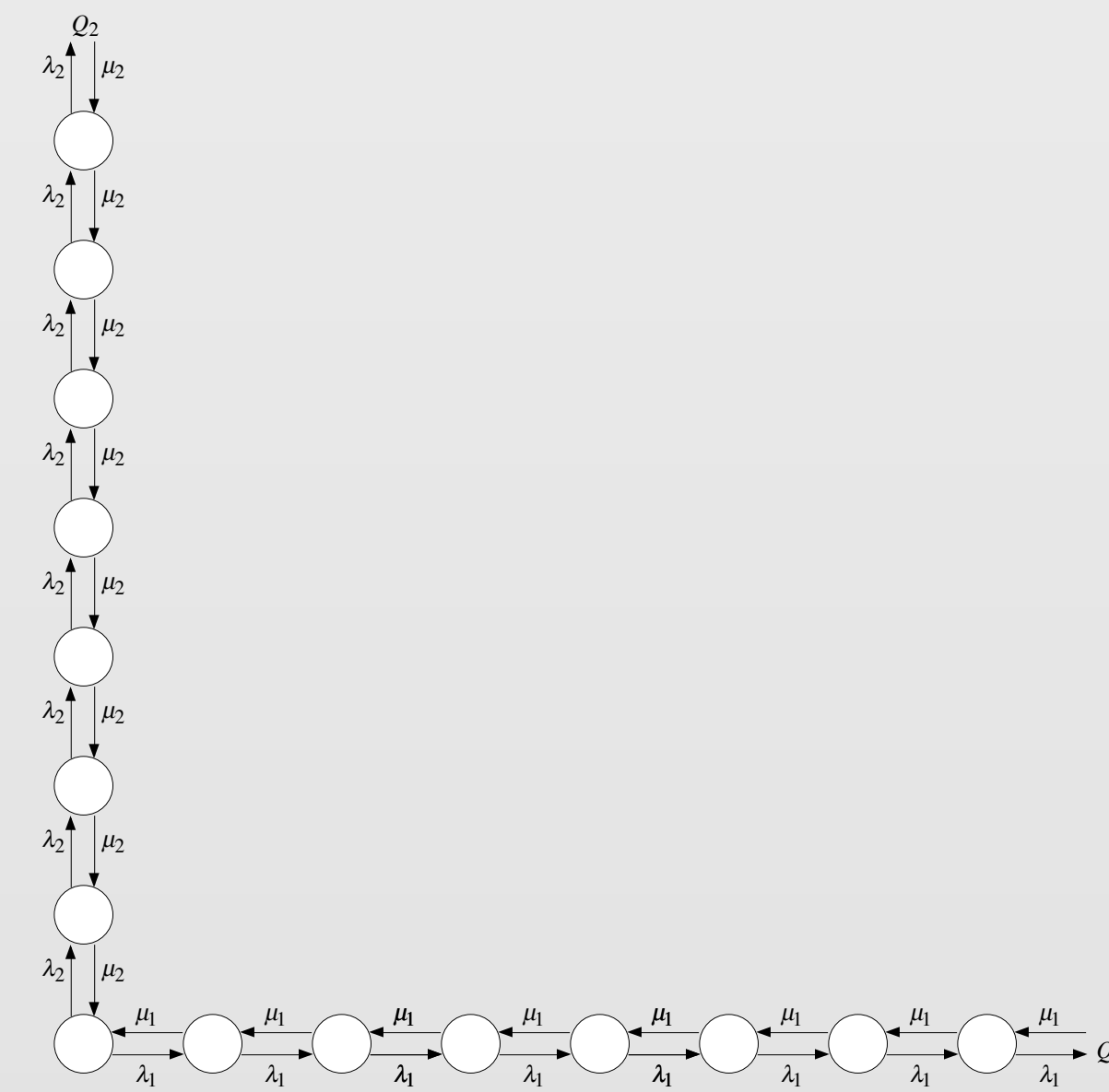
- Output rates for full utilization stable policy:

$$\nu_1 = \frac{\lambda_1 \mu_1 (\lambda_2 - \mu_2)}{\lambda_1 \lambda_2 - \mu_1 \mu_2}, \quad \nu_2 = \frac{\lambda_2 \mu_2 (\lambda_1 - \mu_1)}{\lambda_1 \lambda_2 - \mu_1 \mu_2}$$

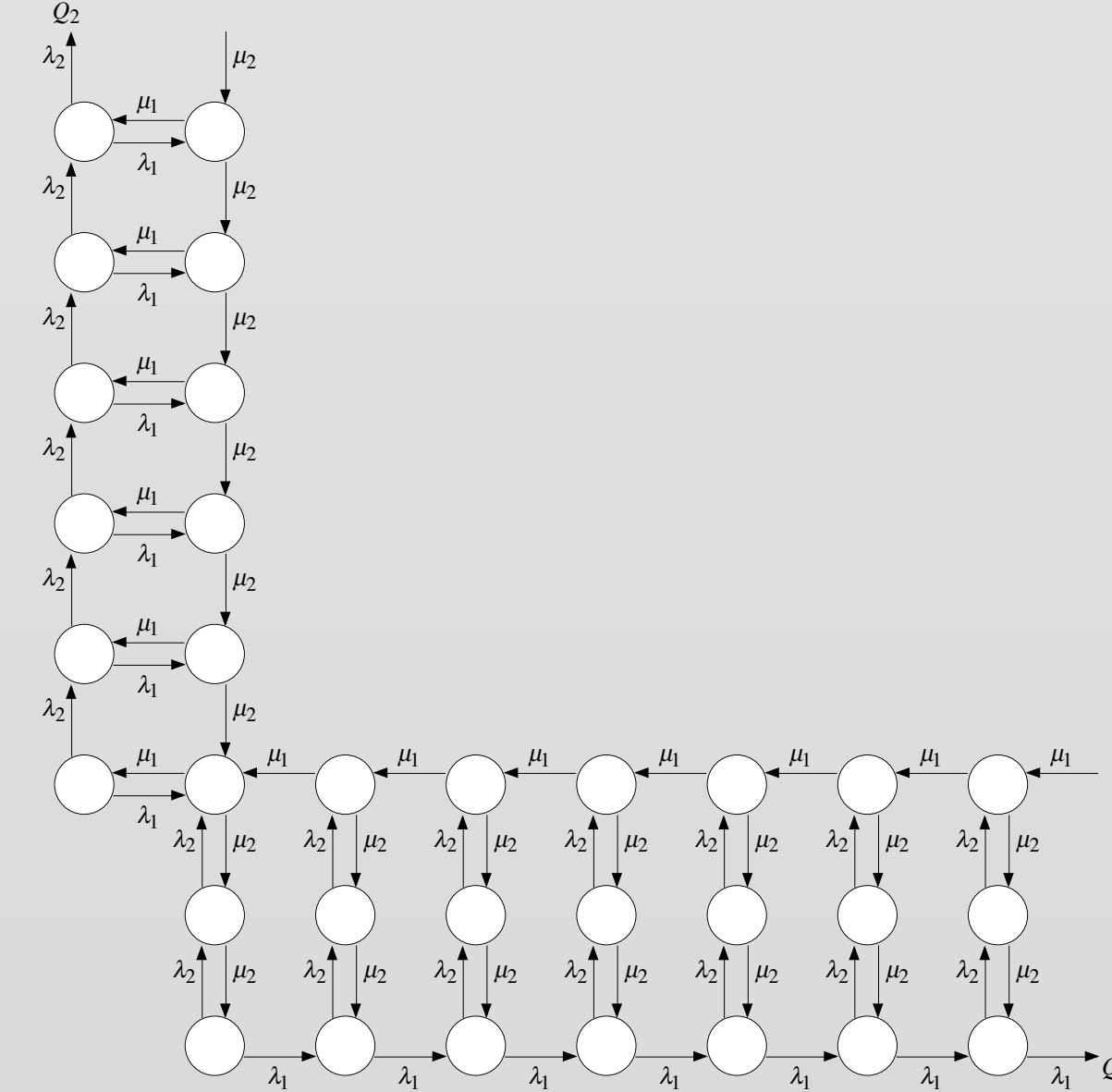
Memoryless Case

- Policy induces a Markov jump process
- Stability analysis for classes of policies
- Stationary distributions for special cases:

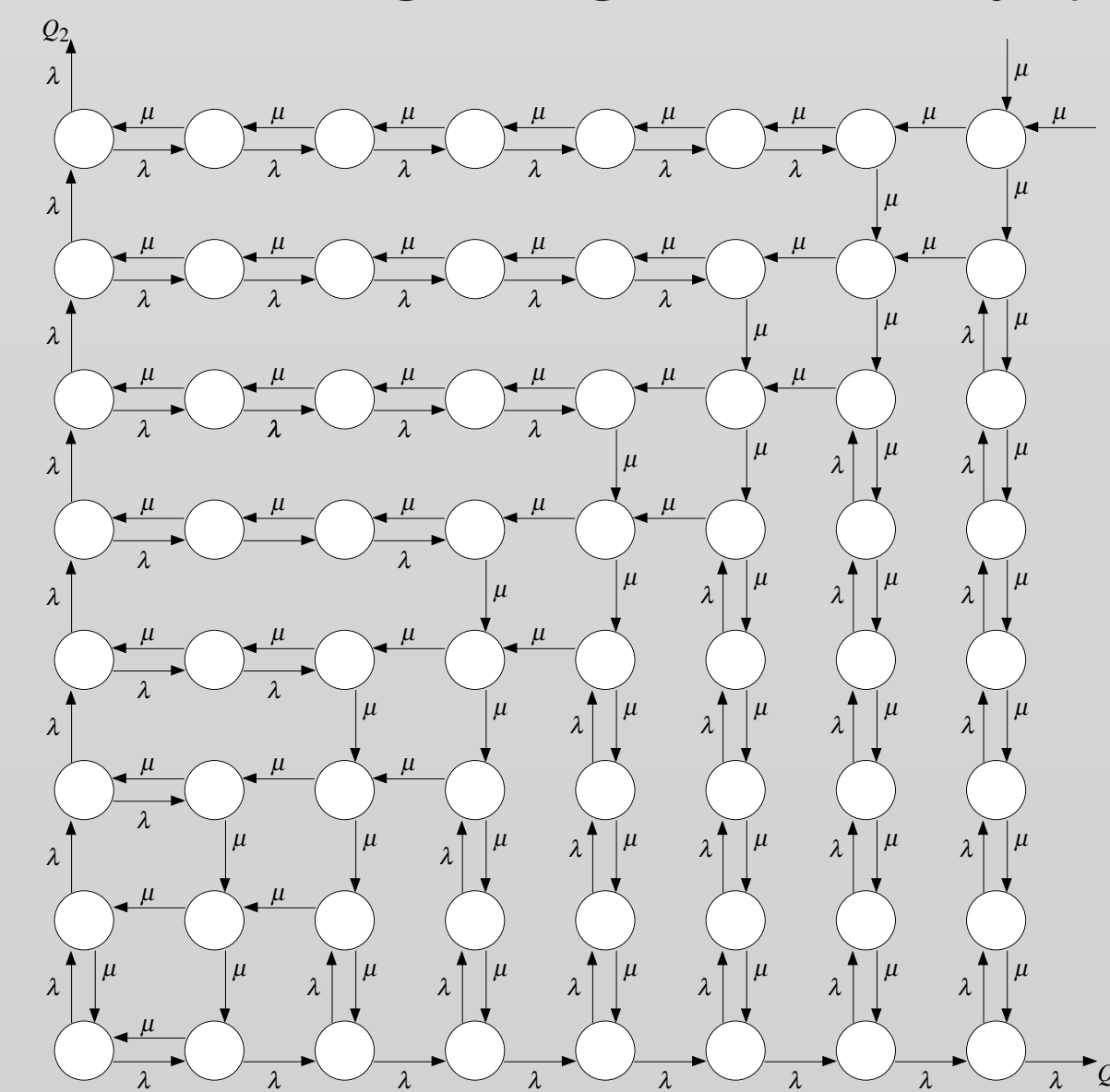
Pull Priority Policy ($\lambda_i < \mu_i$)



Fixed Threshold Policy ($\lambda_i > \mu_i$)



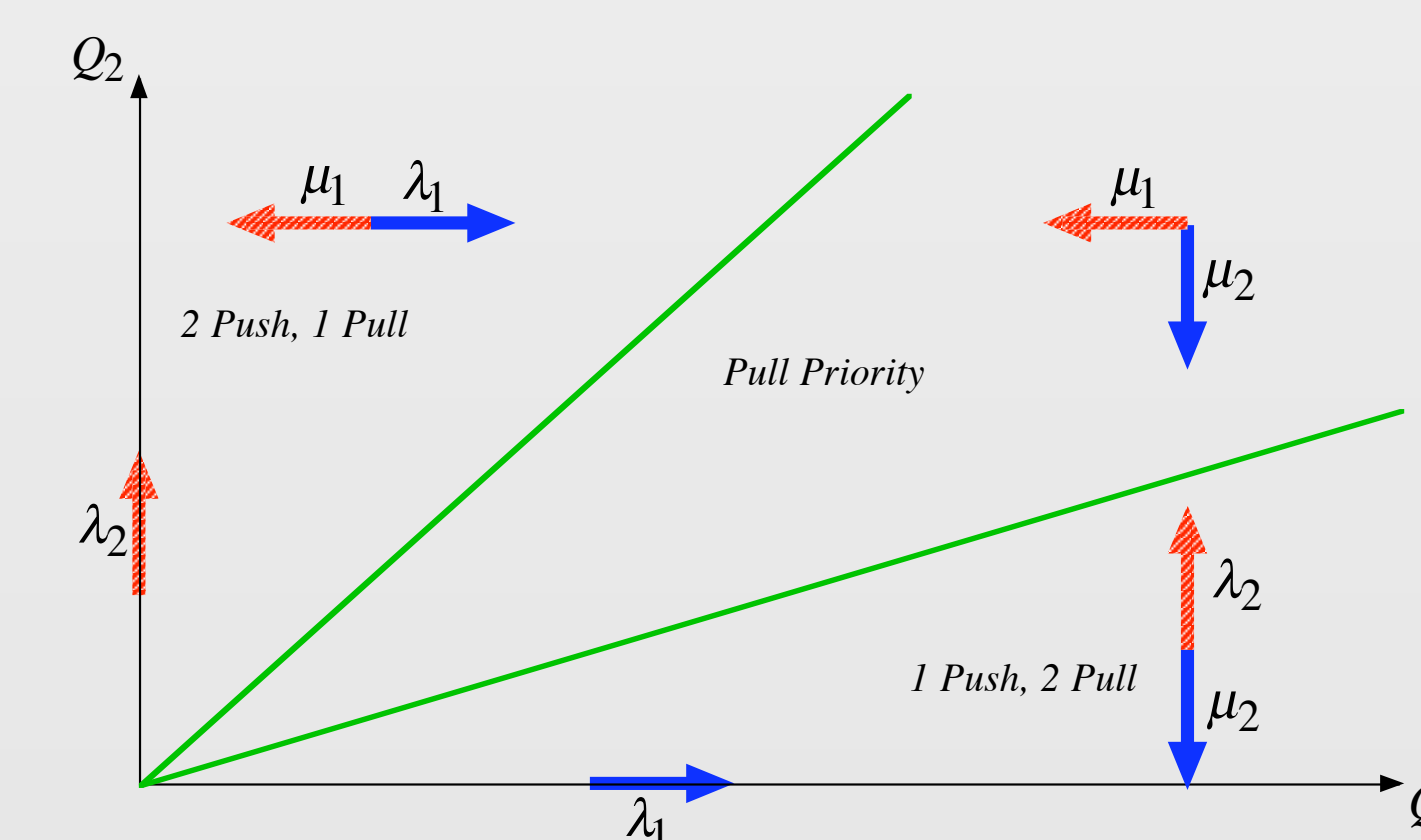
Queue Balancing Diagonal Policy ($\lambda_i > \mu_i$)



General Case

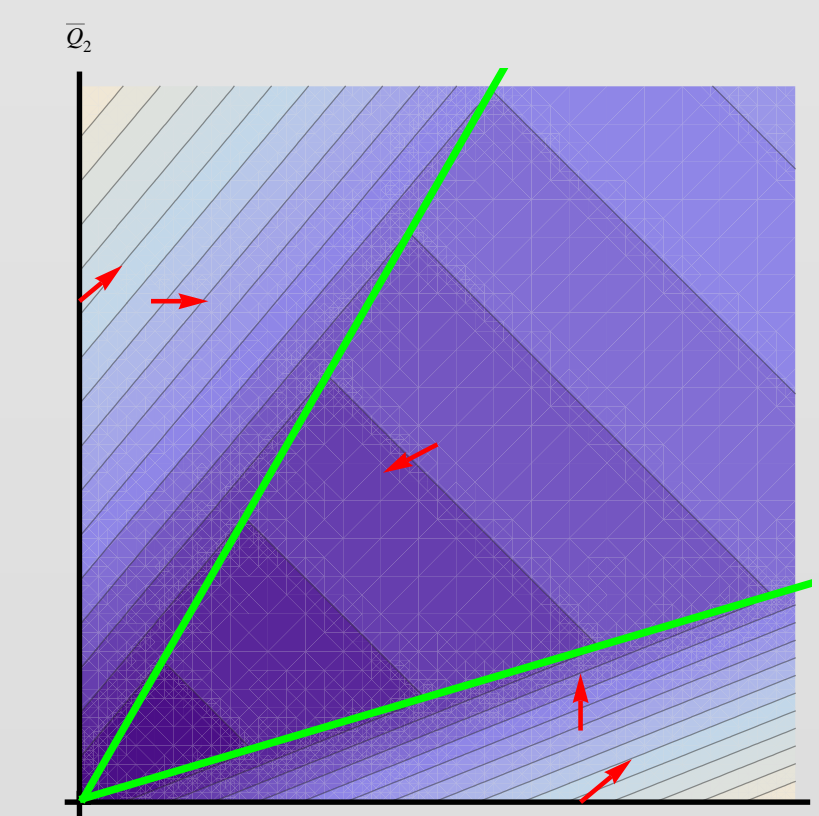
Policies:

- Pull priority policy ($\lambda_i < \mu_i$)
- Linear threshold policy ($\lambda_i > \mu_i$):



Results:

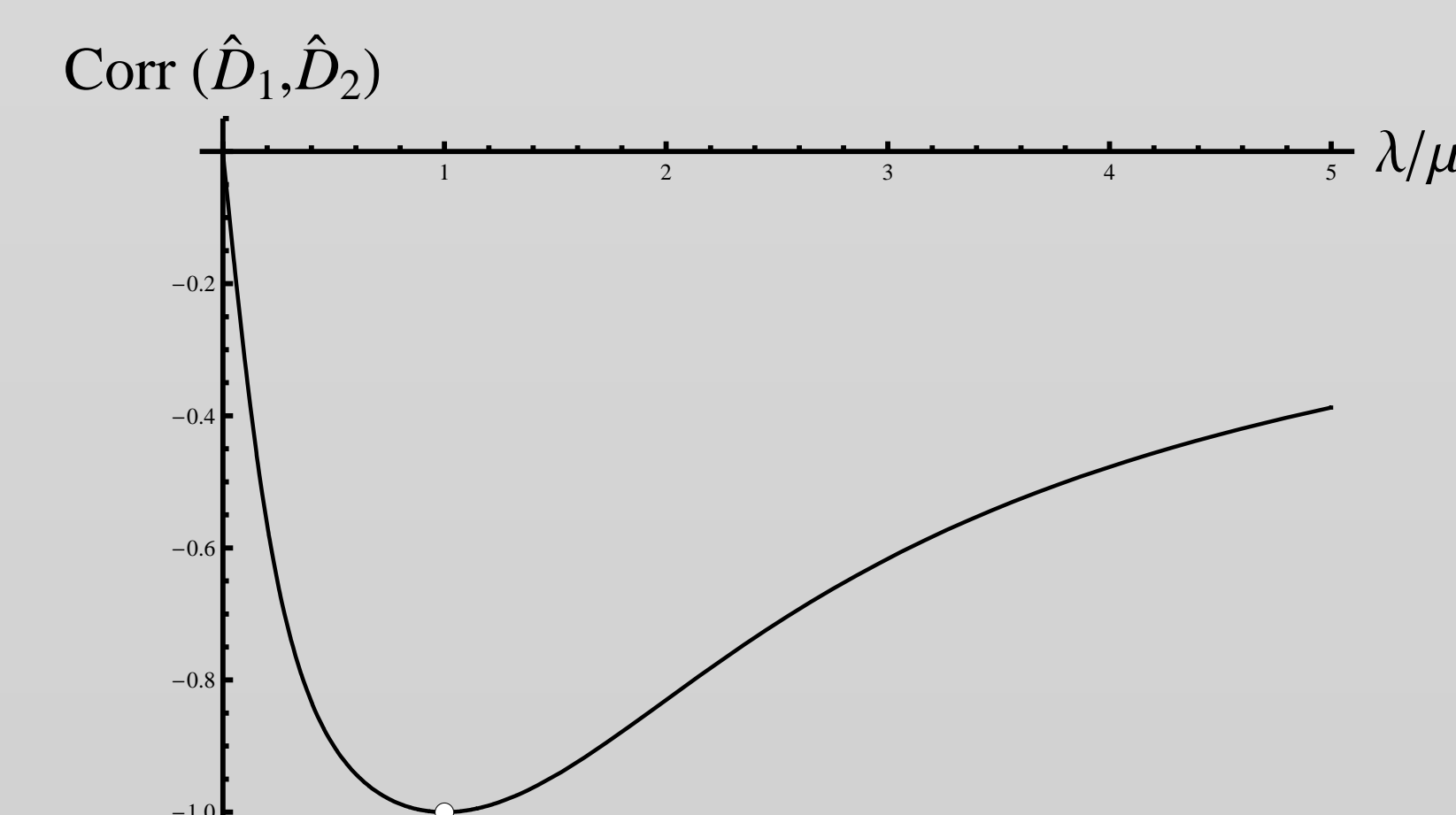
1. Fluid Stability: Fluid models are stable. Lyapounov function (linear threshold policy):



2. Positive Harris Recurrence: Adaptation of Dai's Framework. Modified proof of petiteness of compacts.
3. Diffusion Limits:

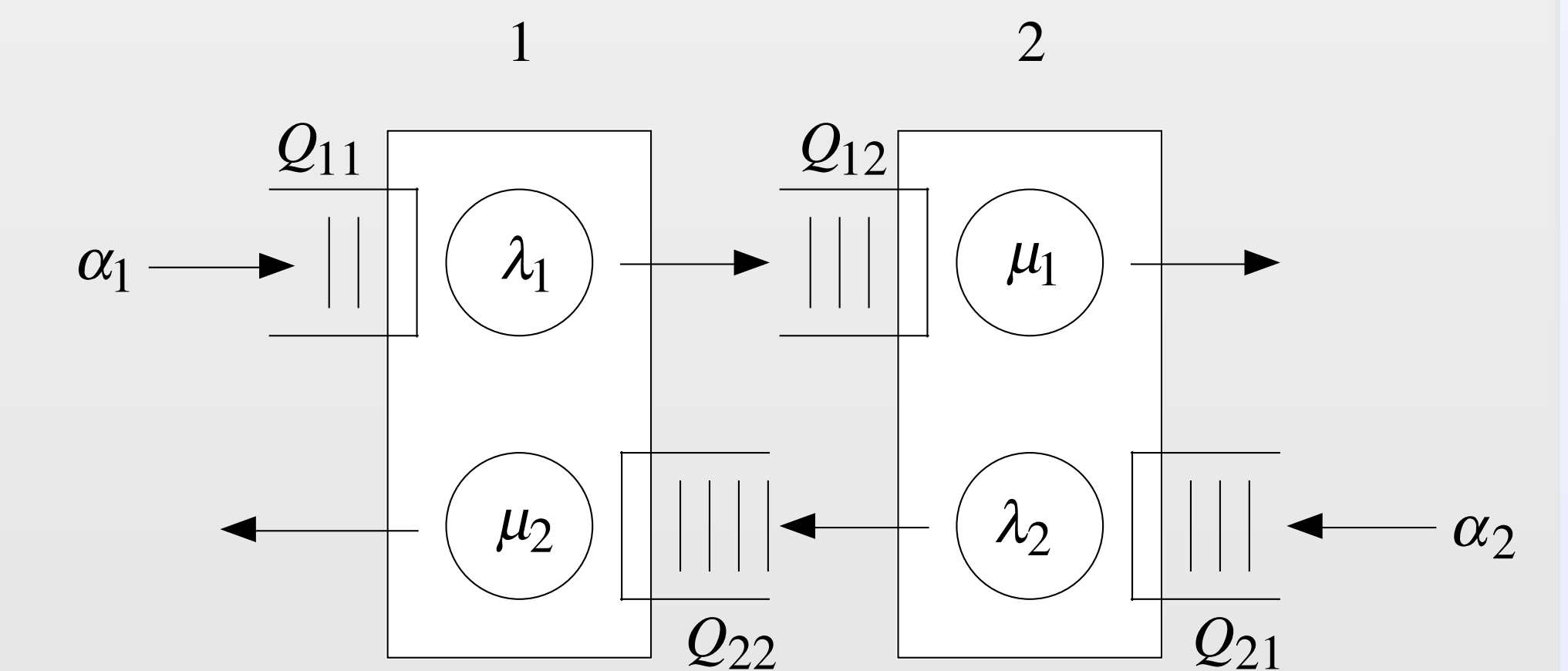
$$\hat{Q}^n(t) \Rightarrow 0, \quad \hat{D}^n \Rightarrow BM(0, \Sigma)$$

Correlation of outputs



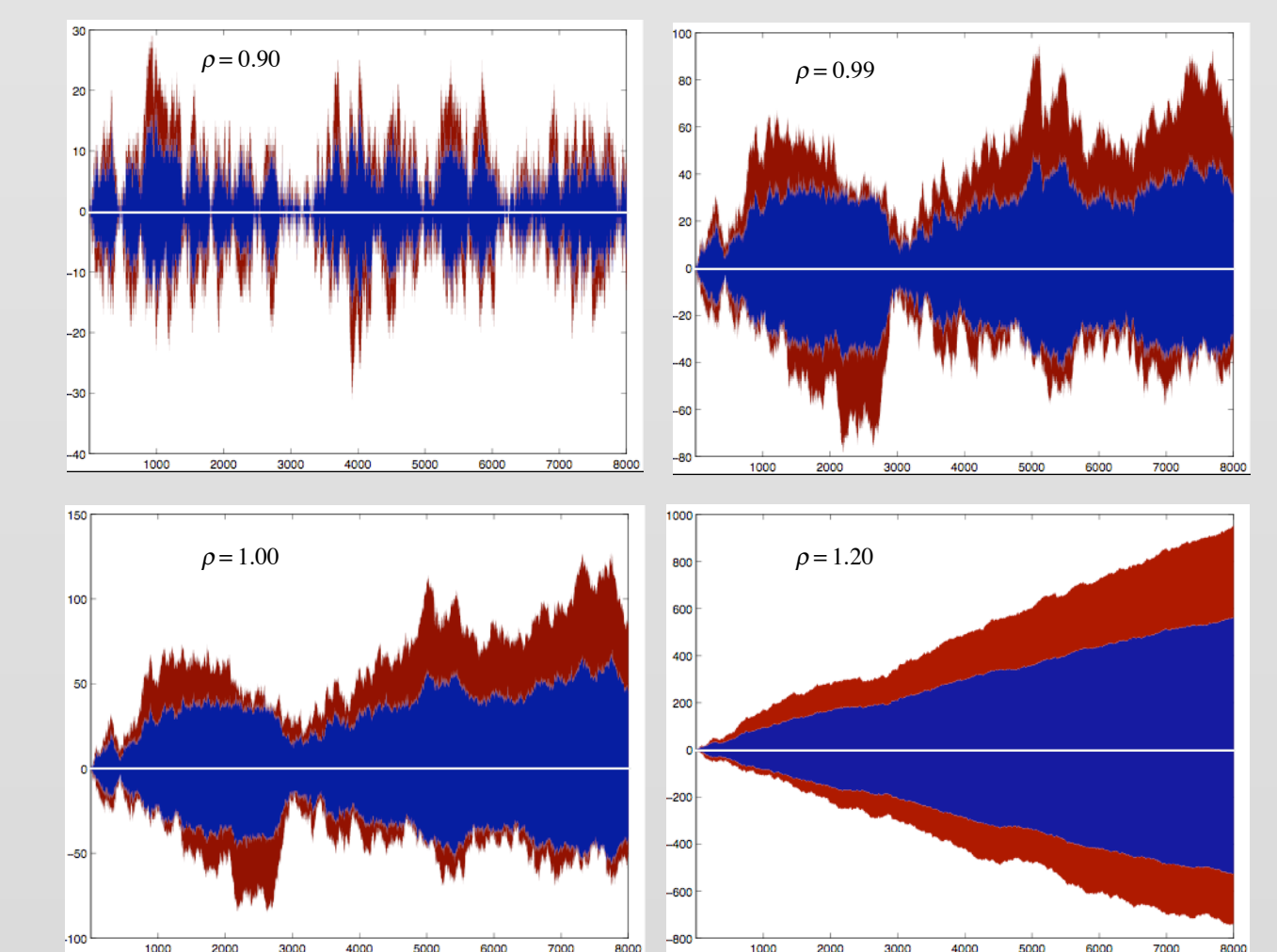
Comparing to KSRS

The Kumar-Seidman-Rybko-Stolyar network:

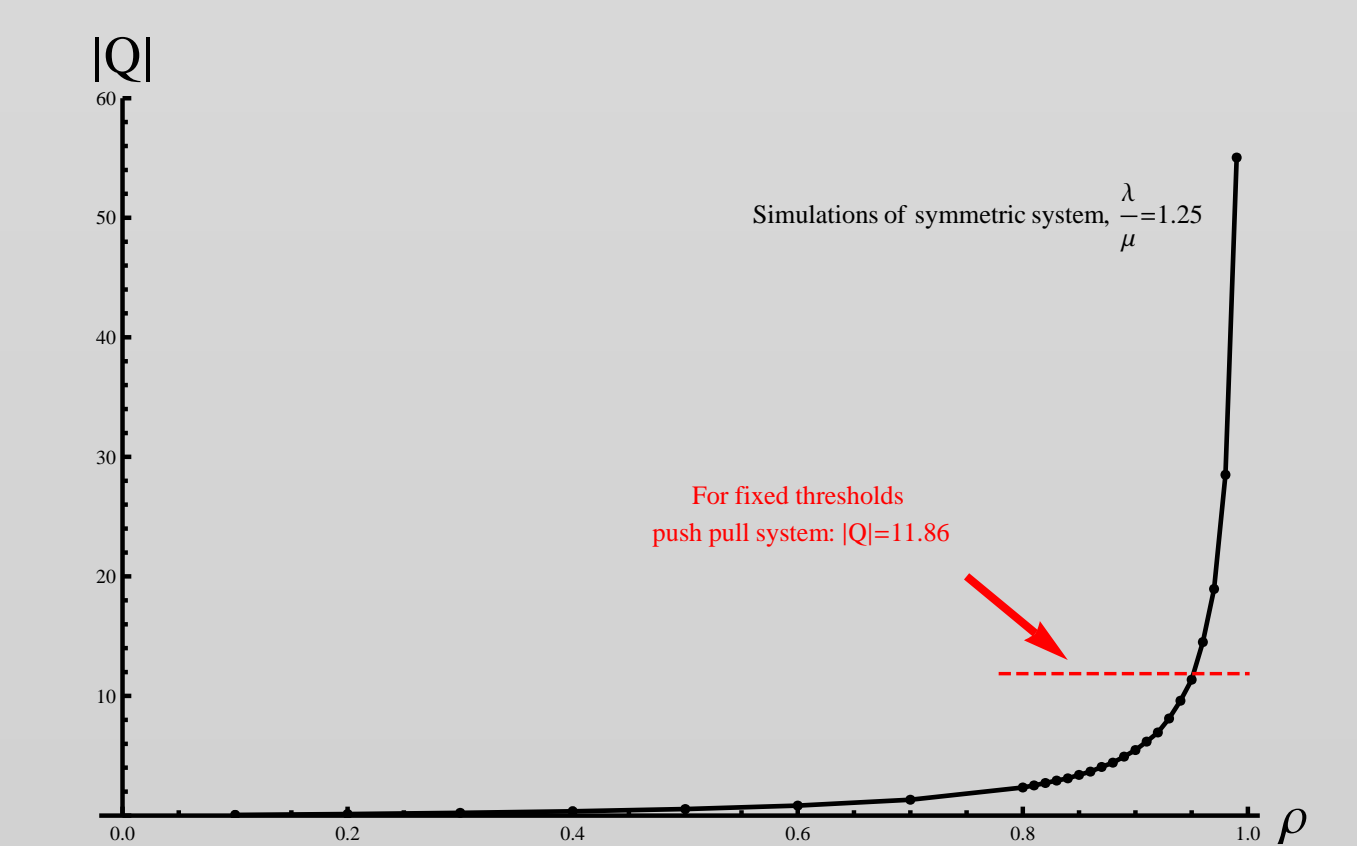


$\alpha_i \nearrow \nu_i$ balanced heavy traffic:

- Maximum Pressure Policy, [1]:



- FBFS (similar to policy of [2]):



References

- [1] Dai, JG and Lin, W. Maximum Pressure Policies in Stochastic Processing Networks. *Operations Research*, 2005.
- [2] Henderson, S.G. and Meyn, S.P. and Tadić, V.B. Performance Evaluation and Policy Selection in Multiclass Networks. *Discrete Event Dynamic Systems*, 2003.